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# Discount for Lack of Marketability

## Job Aid for IRS Valuation Professionals

September 25, 2009

Developed by Engineering/Valuation Program DLOM Team

<sup>Text</sup>  
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## ***Disclaimer***

This job aid is meant to provide information to IRS Valuation Analysts when considering the Discount for Lack of Marketability (DLOM). Always read Section E, Evaluation and Recommendations, in conjunction with Section D, Summary of Approaches to DLOM. . It reviews selected past and existing practices and attempts to provide insight into the strengths and weaknesses of these practices. It is a guide that is not all encompassing and should not be cited, or relied upon, as legal, or any other, precedent. Any opinions expressed are those of the job aid developers not the position of the IRS. The Valuation Analyst must have a clear understanding of the facts and circumstances of each interest to be valued, use professional judgment in choosing a DLOM just as is done for all other parts of a valuation, and apply a reasonableness test. In other words, the Analyst must get behind the data used to support a DLOM choice rather than simply using summary statistics and resulting conclusions developed by somebody else.

The job aid is not an IRS position, and does not make any bright line selections or exclusions as to what approach to DLOM is best in any given set of circumstances—that is up to the Valuation Analyst's professional judgment.

## ***Acknowledgements***

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## **A. Executive Summary**

In June 2008 a team was formed for the purpose of exploring and developing information to assist valuers in the Internal Revenue Service Large and Mid-Size Business (LMSB) Engineering Program in dealing with the Discount for Lack of Marketability (DLOM) as such is used in valuation reports. Among the activities to be undertaken by the team was the clarification of the definition of Discount for Lack of Marketability, exploration of the state of the art in estimating this discount, analysis of current estimating models, review of court commentaries, and documentation of any concerns with the use of the various approaches considered. The team's focus was to identify issues around DLOM and to present techniques to assist valuers in the field. This information should be of value not only to our own personnel but also to our valuation customers.

**Objective:** The team researched the state of art in DLOM starting by defining DLOM and differentiating it from such related areas as Discount for Lack of Liquidity (DLOL) and Discount for Lack of Control (DLOC). We reviewed long-standing methods for estimating DLOM. We explored the models in recent professional journals, discussed the pro's and con's of these models, explored their strengths and weaknesses and looked for elements of reconciliation among the models where possible. As a result of this initial work, the team developed a job aid that addresses the more common approaches being used in the valuation community. Our hope was to provide a quality, timely analysis that will assist employees in the field working DLOM issues.

**Approach:** It is recognized that the DLOM issue is primarily factual in nature. However, it is also recognized that many of the aspects of this issue have been explored by the courts and the courts have defined, in part, what facts may be given weight in determining the DLOM on a given case. Therefore, the LMSB Engineering Program and Estate & Gift Tax Program (E & G) of the Small Business and Self-Employed (SBSE) division are key players in the need for this analysis. Annually, an Estate Tax Attorney compiles a listing of key cases on E & G issues. His willingness to provide a current summary analysis on this topic was vital to the work of our team.

The information provided in this document is thought to address issues on the most current approaches to DLOM. Any model or estimating technique can be misused and abused. The commentary that follows addresses various approaches and models associated with the quantification of a DLOM as of the

## A. Executive Summary

date of this report. Further updates and changes to these models or techniques could render some of these comments obsolete.

**Conclusion:** This Job Aid is meant to provide a background and context for the Discount for Lack of Marketability as such is commonly applied in business valuation analyses and reports. It reviews past and existing practices and attempts to provide insight into the strengths and weaknesses of these practices. It is not meant to provide a cookbook approach to evaluating a marketability discount as proposed by a taxpayer or to setting a proposed marketability discount in the case of an independent governmental appraisal. Nor is it meant to be an IRS position or cited as precedent. Any opinions expressed are suggestions of the job aid developers not the position of the IRS. It is emphasized that, all background and existing practices aside, the establishment of a Discount for Lack of Marketability is a factually intensive endeavor that is heavily dependent upon the experience and capability of the valuator. By bringing the included material together in one document, we are striving to make the job of the IRS valuation analyst easier. We do not mean to provide guidance as to reasonable levels of marketability discounts that would prevail in all situational contexts or to imply that the IRS has any policy per se in the evaluation or the determination of such discounts.

## ***B. Introduction***

The application of the Discount for Lack of Marketability (DLOM) can result in a significant value reduction as compared to the pro rata value of a business interest. Frequently, this discount is the subject of controversy in IRS valuation work, particularly in Estate & Gift Tax cases. Today's valuation practitioners utilize numerous studies, methods and models as the source for DLOM as it is applied to a specific subject interest. These studies, methods and models can be complex, can indicate widely diverse conclusions, and may be appropriate in only certain limited situations. The business valuation profession does not identify acceptable or unacceptable methods for estimating marketability discounts, although some individual practitioners have their own preferences and frequently disagree as to the best approach. Research in DLOM continues for improved data sources and theory. Some of this research is published primarily as an academic pursuit but is untested in practice.

The purpose of this job aid is to assist IRS valuation analysts in their examination of and their independent determination of DLOM and to help them to better understand the numerous available approaches. First, we will identify the current state of DLOM studies and methods—ranging from the SEC Restricted Stock study prepared in 1971 to the Liquistat database announced in 2007. We will endeavor to explain the intent of the approaches most widely relied upon by practicing valuers as to how each estimates DLOM. We will identify the parameters used in a given approach, the strengths and weaknesses of the approach, the view of the valuation community concerning the approach, and what the courts have had to say about the approach, if anything. The job aid also provides initial IDR questions for examination of DLOM and some sample report language for reviewers to consider in situations where it's clear that the approach being used by the taxpayer is in error.

The DLOM Team formed to consider discounts for lack of marketability includes IRS Valuation professionals from across the country. While the team worked together on this project, members developed specific portions:

- Benchmark Approaches
- Securities-Based Approaches
- Analytical Approaches
- Other Approaches



## **C. General Marketability Discount Information**

### **1. Marketability Defined**

Marketability is defined in the *International Glossary of Business Valuation Terms* as “the ability to quickly convert property to cash at minimal cost”<sup>1</sup>. Some texts go on to add “with a high degree of certainty of realizing the anticipated amount of proceeds”.<sup>2</sup>

A Discount for Lack of Marketability (DLOM) is “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.”<sup>3</sup>

Given two identical business interests, a higher price will be paid by investors in the market for the business interest that can be converted to cash most rapidly, without risk of loss in value. An example is publicly-traded stock on the New York Stock Exchange, where the owner can order the sale and the proceeds are deposited in a bank account in three days.

In the alternative, a lesser price is expected for the business interest that cannot be quickly sold and converted to cash. A primary concern driving this price reduction is that, over the uncertain time frame required to complete the sale, the final sale price becomes less certain and with it a decline in value is quite possible. Accordingly, a prudent buyer would want a discount for acquiring such an interest to protect against value loss in a future sale scenario.

What to remember about DLOM:

- DLOM is appropriate when the subject interest is non-marketable, yet the prior steps in the valuation process result in a marketable value.
- DLOM is not appropriate if the prior valuation process has already taken marketability concerns into consideration.
- DLOM is applied after the minority interest discount or control premium where such is appropriate to a valuation problem.
- DLOM should be determined on its own factors and not combined with other discounts.

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<sup>1</sup> *International Glossary of Business Valuation Terms*, as adopted in 2001 by American Institute of Certified Public Accountants, American Society of Appraisers, Canadian Institute of Chartered Business Valuators, National Association of Certified Valuation Analysts, and The Institute of Business Appraisers.

<sup>2</sup> Shannon P. Pratt, Alina V. Niculita, *Valuing a Business, The Analysis and Appraisal of Closely Held Businesses*, 5<sup>th</sup> ed (New York: McGraw Hill, 2008), p.39.

<sup>3</sup> *International Glossary*.

## C. General DLOM Information

### Marketability vs. Liquidity

What is liquidity? Liquidity is the ability to quickly convert property to cash or pay a liability.<sup>4</sup>

Said another way, Liquidity is the ability to readily convert an asset, business, business ownership interest or security into cash without significant loss of principal. Compare Liquidity to the definition of Marketability: the capability and ease of transfer or salability of an asset, business, business ownership interest or security.

A Discount for Lack of Liquidity (DLOL) is an amount or percentage deducted from the value of an ownership interest to reflect the relative inability to quickly convert property to cash.

How does Liquidity differ from Marketability? These terms are often used interchangeably, although there is a technical distinction between them. Marketability indicates the fact of “Salability”, while Liquidity indicates how fast that sale can occur at the current price.

- If it's liquid, it's marketable
- If it's non-marketable, it's illiquid
- Being illiquid does not necessary mean non-marketable – it may still be sellable but not quickly or without loss of value

Some distinguish marketability and liquidity as follows: “...with marketability focusing on finding the appropriate market, preparing the property for sale and executing the trade, and liquidity focusing on realizing cash proceeds.”<sup>5</sup>

We define liquidity here because some of the studies or methods discussed in the job aid make a distinction between DLOM and DLOL.

## 2. Factors Influencing Marketability Identified

Factors that can have an influence on marketability are numerous. A prominent Tax Court case set forth factors for consideration of DLOM. The Mandelbaum Factors were set out in *Mandelbaum v. Comm.*, TC Memo 1995-255 (1995), with the opinion written by Judge Laro. The factors and the analysis that go with them are considered by many valuers to form a good conceptual basis for thinking about and quantifying DLOM.

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<sup>4</sup> *International Glossary*.

<sup>5</sup> Pratt, *Valuing a Business*, p.39.

## C. General DLOM Information

Some common factors that have been identified in various studies as impacting marketability are listed below and are modeled after the Mandelbaum factors. The first set of factors relate to the characteristics of the subject company. The second set of factors relate to the characteristics of the subject interest.

### Subject Company Factors

- Value of subject corporation's privately traded securities vs. its publicly traded securities (or, if the subject corporation does not have stock that is traded both publicly and privately, the cost of a similar corporation's public and private stock)
- Dividend-paying (or distribution) ability and history
- Dividend yield
- Attractiveness of subject business
- Attractiveness of subject industry
- Prospects for a sale or public offering of the company
- Number of identifiable buyers
- Attributes of controlling shareholder, if any
- Availability of access to information or reliability of that information
- Management
- Earnings levels
- Revenue levels
- Book to market value ratios
- Information requirements
- Ownership concentration effects
- Financial condition
- Percent of shares held by insiders
- Percent of shares held by institutions
- Percent of independent directors
- Listing on a major exchange
- Active vs. passive investors
- Registration costs
- Availability of hedging opportunities
- Market capitalization rank
- Business risk

### Subject Interest Factors

- Restrictive transfer provisions
- Length of the restriction period
- Length of expected holding period
- Offering size as a % of total shares outstanding
- Registered vs. unregistered

## C. General DLOM Information

- General economic conditions
- Prevailing stock market conditions
- Volatility of stock

In *Mandelbaum*, the landmark case for marketability, Judge Laro set out various factors to be considered in determining DLOM.

...Ascertaining the appropriate discount for limited marketability is a factual determination. Critical to this determination is an appreciation of the fundamental elements of value that are used by an investor in making his or her investment decision. A nonexclusive list of these factors includes: (1) The value of the subject corporation's privately traded securities vis-a-vis its publicly traded securities (or, if the subject corporation does not have stock that is traded both publicly and privately, the cost of a similar corporation's public and private stock); (2) an analysis of the subject corporation's financial statements; (3) the corporation's dividend-paying capacity, its history of paying dividends, and the amount of its prior dividends; (4) the nature of the corporation, its history, its position in the industry, and its economic outlook; (5) the corporation's management; (6) the degree of control transferred with the block of stock to be valued; (7) any restriction on the transferability of the corporation's stock; (8) the period of time for which an investor must hold the subject stock to realize a sufficient profit; (9) the corporation's redemption policy; and (10) the cost of effectuating a public offering of the stock to be valued, e.g., legal, accounting, and underwriting fees.<sup>6</sup>

These “Mandlebaum Factors” are often used by valuers and are regularly seen in court cases where DLOM is an issue. For more on Mandelbaum, refer to the Benchmark studies at D.1.e. in this job aide.

### 3. Willing Seller Consideration

In determining/justifying marketability discounts, many appraisers only consider the willing buyer. However, common sense and the courts have emphasized that a willing seller must also be considered. In considering the market for a subject interest, the applicable market in which a hypothetical willing buyer may be found need not be one that includes the general public. The courts have determined that it is sufficient if there are potential buyers among those closely connected with a corporation.

- In *Luce v. US*, 4 Cl. Ct. 220-221 & 222 (53 AFTR 2d 84-1565, 84-1 USTC

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<sup>6</sup> *Mandelbaum v. Comm.*, T.C. Memo 1995-255, 36.

## C. General DLOM Information

13549), in addition to the corporation itself and its controlling stockholders there was a further market for the shares among the other managerial employees. Thus there was no need for a 30% discount in order for the hypothetical seller to find a willing buyer.

- In *Rothgery v. US*, 201 Ct. Cl. 183,189, the court held that the decedent's son would have been a willing buyer of the shares from any hypothetical seller and that this was a market sufficient to negate any need for a discount to sell the shares.
- In *Couzens v. CIR*, 11 BTA 1164 (1928), the court stated "we do not construe a fair market as meaning that the whole world must be a potential buyer, but only that there are sufficient available persons able to buy to assure a fair and reasonable price in light of the circumstances affecting value".
- In *Estate of Jephson v. Comm.*, 87 T.C. 297, (a case involving cash and marketable securities held in a partnership) the court stated that "In our opinion, neither the decedent nor her estate nor a hypothetical seller would have sold the stock of either company for less than that which could have been realized through liquidation. We further believe that a hypothetical purchaser would be willing to pay such an amount. The hypothetical purchaser, by purchasing the companies, would have brokerage fees that otherwise would have to be paid to acquire approximately \$9 million of marketable securities."
- Also see: *Estate of Goldstein v. Comm.*, 33 T.C. 1932,1037 (1960); *Smith v. Comm.*, 46 BTA 340-41 (1942); and *Worcester County Trust Co. v. Comm.*, 134 F.2d 578 (1st Cir. 1943).

## 4. Marketability of Minority vs. Controlling Interests

There is little dispute that minority interests in non-publicly traded entities lose value due to lack of marketability. However, the issue of applying a discount for lack of marketability to a controlling interest is a controversial issue<sup>7</sup> amongst valuers. Some believe that there should be little or no discount for lack of marketability on a controlling interest, while others believe there should be a discount applied. Most agree that any marketability discount for a controlling interest should be less than the discount for a minority interest in the same entity.

The controlling interest owner will be able to sell his or her business interest in one of two ways: a public offering or a private sale. As Pratt discusses in Valuing a Business, the following factors will have to be considered:

- Uncertain time horizon to complete the offering or sale;

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<sup>7</sup> Pratt, Shannon, Business Valuation Discounts and Premiums, (NY: John Wiley & Sons, 2001), p.167.

## C. General DLOM Information

- Costs to prepare and execute the offering or sale (legal, accounting, administrative, brokerage)
- Risk as to eventual sales price
- Non-cash and deferred transaction proceeds
- Inability to hypothecate<sup>8</sup>

Because of these considerations, the controlling interest owner may not be able to sell the interest quickly or with certainty as to the ultimate sales price.

Therefore, it follows that the controlling interest may not be fully marketable.

Among valuers who apply DLOM to controlling interests, it is generally agreed that DLOM of a controlling interest is less than that for a minority interest.

Court cases where DLOM was considered and allowed on a controlling interest:

- *Estate of Andrews v. Comm.*, 79 TC 938 (1982)
- *Estate of Simpson v. Comm.*, TCM 1994-207
- *Gray v. Comm.*, TCM 1997-67

## 5. Sample Initial IDR Items on Marketability

The evaluation of the appropriateness of a discount for lack of marketability requires the collection and analysis of a substantial amount of information about the entity involved and the subject interest in that entity whose marketability is being considered. We provide below a list of typical inquiry areas that can be put forth in Information Document Requests toward the end of collecting such information. The bracketed notes below each item offer commentary about that item's relevance in evaluating its contribution to the lack of marketability and/or lack of liquidity determination.

- a. History of dividend payments [cash dividends are a “liquid” return on investment, which might lower lack of marketability risk]
- b. Salaries and bonuses paid to the Officers of the company, over the five years leading up to the valuation date [especially in companies that don't pay dividends, Officers' compensation can provide cash flow to shareholders, which might lower lack of marketability risk]
- c. Compensation and/or fees paid to the Directors of the company, over the five years leading up to the valuation date [especially in companies that don't pay dividends, Directors' fees can provide cash flow to shareholders, which might lower lack of marketability risk]

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<sup>8</sup> Pratt, Reilly, Schweihs, Valuing a Business, Fourth Edition., p.413.

## C. General DLOM Information

- d. List of all marketable securities (description, number, cost value) shown on the latest financial statements [cash-equivalent securities might lower liquidity risk on a company-wide basis]
- e. List of all non-marketable securities and investments (description, number, cost value) shown on the latest financial statements [can provide information on how long it might take to liquidate non-marketable assets]
- f. Breakdown of adjusted cost basis for each of the marketable and Non-marketable assets owned by the company on the valuation date [can provide information on built-in capital gains tax expense to liquidate the company]
- g. Indicate if the adjusted cost basis of any of the company's marketable or non-marketable assets reflects a carry-over cost basis, pursuant to a Section 1031 (or similar type) tax-deferred exchange [can provide information on whether the company pursues available tax-deferral strategies]
- h. Current list of shareholders/partners showing the name of each shareholder/partner and the class and number of shares owned by each shareholder as of the valuation date [can provide information on relative ownership distribution and total number of shareholders]
- i. Copies of notes receivable (and/or notes payable) between the company and any shareholders, over the five years leading up to the valuation date [loans to/from shareholders might be relevant to evaluating lack of marketability risk]
- j. Company articles of incorporation and amendments, by-laws and amendments or partnership agreements and amendments [by-laws might address restrictions or procedures for transfer of shares]
- k. Copy of all shareholder agreements (such as buy/sell agreements, stock option agreements stock purchase agreements, etc.) that have been in effect during the five years prior to the valuation date [shareholder agreements might address restrictions or procedures for transfer of shares]
- l. All documents pertaining to any sale of the company, a division or unit of the company, or shares (interests) in the company during the five years

## C. General DLOM Information

prior to the valuation date [recent sales/transfers might be might be relevant to evaluating lack of marketability risk]

- m. Board of Directors Meeting Minutes, for five years leading up to valuation date [Board meetings might address shareholder requests for sale/transfer of shares]
- n. Complete financial statements of the company for the five fiscal or calendar years prior to the valuation date, including balance sheets, income statements and cash flow statements [can provide additional information for evaluating lack of marketability risk]
- o. Complete income tax returns for the five fiscal or calendar years prior to the valuation date, including any audit adjustments [tax returns might include details that are not stated within the regular financial statements]
- p. Brief history and/or description of the company or the company's business (may already be included in an appraisal report) [can provide additional information for evaluating lack of marketability risk]
- q. Brief statement of duties of subject shareholder's participation in company operations [can provide additional information for evaluating lack of marketability risk]



## **D. *Summary of Approaches to DLOM***

There have been numerous approaches taken by researchers and practitioners for determining the appropriateness of allowing a discount for lack of marketability in the valuation of a business interest and in estimating the magnitude of such a discount. For discussion purposes, we have classified these approaches into four categories:

1. Benchmark study approaches,
2. Security-based approaches,
3. Analytical approaches and
4. Other approaches.

These categories are discussed individually in this section of the job aid.

### **1. Benchmark Approaches**

The so-called benchmark study approaches come in two primary forms – those that estimate appropriate DLOM amounts based on restricted stocks and those that base the DLOM estimate on Initial Public Offering (IPO) pricing as compared to the price of earlier privately traded transactions. There are then certain derivative approaches that have spun out of the basic approaches. We start our discussion with the first of the primary categories -- restricted stocks. We then cover Pre-IPO studies and conclude this section with brief discussions of certain approaches derived from the benchmarks.

#### **a) Restricted Stock Studies**

##### **Background**

Restricted stock<sup>9</sup> has been used over many years by members of the business valuation community to quantify the discount for lack of marketability. The restricted stock studies have been cited and analyzed in numerous court decisions, sometimes with favorable consideration by the

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<sup>9</sup> According to the Securities Act of 1933 (Section 230.144), restricted stock is unregistered common stock of a corporation identical in every respect to its publicly traded shares, except that it has not been registered, and is therefore, not freely tradable. Because the holder of restricted common stock is prohibited from selling any of the stock for full year (1997-2008, thereafter holding period is six months) and has additional constraints on the amounts that may be sold for an additional year, the restricted stock is significantly less liquid (and therefore less valuable) than its unrestricted counterpart.

## D. Summary of DLOM Studies/Methods Benchmark

court and sometimes with no real consideration at all. The premise behind the restricted stock studies is that the effect of lack of marketability can be quantified by comparing the sale price of publicly traded shares to the sale price of so-called restricted shares of the same company that are identical in all rights and powers except for their ability to be freely marketed. This restriction on marketability for the restricted shares is time-limited but does act to affect the ability of the holder to trade the shares during the restriction period. Under SEC rules this restriction period has been either one or two years depending upon the time of issuance of the shares.

Many different researchers have collected data on restricted stocks and have compared them to their publicly traded counterparts beginning in 1966. The studies conducted have included various time periods for collecting the data and have generated a number of summary statistics to describe the data analyzed.

In recent years, as the discipline of business valuation has continued to evolve, the valuation communities and the courts have taken an increasingly critical view of simply beginning with a summary statistic from a group of studies and going from there, either by accepting the statistic as is or adjusting it without a believable explanation. Attention has turned instead to getting behind the data itself and deriving an appropriate discount from the data as such relates to the case at hand. This evolution needs to be understood by the valuator and duly considered in using restricted stock studies as an approach to DLOM.

Restricted stock studies are published, empirical studies, the most often cited of which are indicated below. These studies analyze market data during 1966-1996 in which public company stock prices were viewed relative to the prices of such companies' restricted stock issues. Because SEC restricted stock rules prior to 1997 required investors to hold such stock for at least two years (and after 1997 to the present for at least one year), the difference between prices at which restricted stocks were issued relative to freely traded stocks of the same company are considered a proxy for a marketability discount for non-publicly traded stock. This analogy is considered to be appropriate since non-publicly traded stocks are also limited in their immediate marketability. A summary of these studies is provided on the next page.

## D. Summary of DLOM Studies/Methods Benchmark

<b>Restricted Stock Studies Attempting to Measure the Marketability Discount for Private Firms</b>		
<b>Empirical Study (full citation below)</b>	<b>Time Period Covered</b>	<b>Average Discount</b>
SEC overall average (a)	1/66 – 6/69	25.8
SEC nonreporting OTC companies (a)	1/66 – 6/69	32.6
Gelman (b)	1/68 – 12/70	33.0
Trout (c)	1/68 – 12/72	33.5
Moroney (d)	1/69 – 12/72	35.6
Maher (e)	1/69 – 12/73	35.4
Standard Research Consultants (f)	10/78 – 6/82	45.0 (median)
Willamette Management Associates (g)	1981 – 1984	31.2 (median)
Silber (h)	1/81 – 12/88	33.8
FMV Opinions, Inc. (i)	1/79 – 4/92	23.0
Management Planning, Inc (j)	1/80 – 12/96	27.1
Bruce Johnson Study (k)	1/91 – 12/95	20.0
Columbia Financial Advisors (l)	1/96 – 4/97	21.0
Columbia Financial Advisors (l)	5/97 – 12/98	13.0
<p>a) Discounts Involved in Purchases of Common Stock (1966-1969), Institutional Investor Study Report of the Securities and Exchange Commission, H.R. Do. No. 92-64, Part 5, 92<sup>nd</sup> Congress, 1<sup>st</sup> Session, 1971, 2444- 2456.</p> <p>b) Gelman, Milton, An Economist Financial Analyst's Approach to Valuing Stock of a Closely Held Company, Journal of Taxation, June 1972, 353-354.</p> <p>c) Trout, Robert R., Estimation of the Discount Associated with the Transfer of Restricted Securities, Taxes, June 1997, 381-384.</p> <p>d) Moroney, Robert E., Most Courts Overvalue Closely Held Stocks, Taxes, March 1993, 144-154.</p> <p>e) Maher, Michael J., Discounts for Lack-of-marketability for Closely Held Business Interests, Taxes, September 1976, 562-71.</p> <p>f) Pittock, William F., and Stryker, Charles H., Revenue Ruling 77-287 Revisited, SRC Quarterly Reports, Spring 1983.</p> <p>g) Willamette Management Associates study (unpublished)</p> <p>h) Silber, William L., Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices, Financial Analysts Journal, July-August 1991, 60-64.</p> <p>i) Hall, Lance S., and Timothy C. Polacek, "Strategies for Obtaining the Largest Valuation Discounts," Estate Planning, January/February 1994. pp. 38-44.</p> <p>j) Oliver, Robert P. and Roy H Meyers, "Discounts Seen in Private Placements of Restricted Stock: The Management Planning, Inc., Long-Term Study (1980-1996)" (Chapter 5) in Robert F. Reilly and Robert P. Schweihs, eds, The Handbook of Advanced Business Valuations (New York: McGraw-Hill, 2000).</p> <p>k) Johnson, Bruce, "Restricted Stock Discounts, 1991-95", Shannon Pratt's Business Valuation Update, Vol. 5, No. 3, March 1999, pp. 1-3. "Quantitative Support for Discounts for Lack of Marketability." Business Valuation Review, December, 1999, pp. 152-155.</p> <p>l) CFAI Study, Aschwald, Kathryn F., "Restricted Stock Discounts Decline as Result of 1-Year Holding Period – Studies After 1990 'No Longer Relevant' for Lack of Marketability Discounts", SHANNON PRATT'S BUSINESS VALUATION UPDATE, Vol. 6, No. 5, May 2000, pp. 1-5.</p>		

## D. Summary of DLOM Studies/Methods Benchmark

As can be seen from this data, the measures of central tendencies for these various studies would imply DLOM amounts of from a low of 13% to somewhere in the vicinity of the mid-40% decile. This is a wide range in terms of central tendency and indicates the probability of a much wider range across the individual data points. Further, the sample sizes in these studies are small, most involving less than 100 individual data points such that the reliability of the summary statistics is subject to considerable data variation. This factor emphasizes the need to get into the data itself instead of staying at the summary level.

### Summary:

- Authors of restricted stock studies have examined transactions in the shares of public and private companies.
- Restricted shares have some form of agreed upon or legal restrictions related to marketability.
- The studies exhibit average means and medians of 31.4% and 33%, therefore many analysts use a discount of about 35% or attach a subjective premium to the average discount to account for the perceived greater illiquidity of a private company's stock versus the restricted stock.
- The DLOM concluded by the more recent restricted stock studies are smaller than the DLOM concluded by the older restricted stock studies. One explanation for this phenomenon is the increase in volume of privately placed stock under Securities and Exchange Commission (SEC) Rule 144(a) in the past several years. Also, a change in the minimum investment holding period required by the SEC under Rule 144 from two years to only one year took place as of April 29, 1997.
- Effective February 15, 2008 the SEC changed Rule 144 by shortening the holding period even further for restricted securities for **small companies**.
- The key to this DLOM approach is the importance of understanding the various marketability studies, how they relate to the subject interest being valued, and whether the ultimate marketability discount that is reasonable for the situation is below, equal to, or above the discounts (or range of discounts) suggested by the studies.

### Areas of Focus

## D. Summary of DLOM Studies/Methods Benchmark

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

- Has Taxpayer's appraiser considered separation of "lack of marketability" from other effects (e.g. blockage) that might be contributing to discounts observed in the Restricted Stock Studies data?
- Has Taxpayer's appraiser addressed variance and/or range of discounts observed in the Restricted Stock Studies data?
- On what basis has Taxpayer's appraiser determined that any particular "average" or "median" discount from the Restricted Stock Studies data is applicable to the subject company?
- On what basis has Taxpayer's appraiser adjusted the average or median discount data for factors applicable to the subject company?
- If Taxpayer's appraiser is using specific restricted stock transactions from a database, on what basis has Taxpayer's appraiser estimated those particular restricted stock transactions to be applicable to the subject company?

### Strengths

- The advantage of using restricted stock studies is that the stock is identical to its freely traded counterpart, except for the duration of the resale restriction, and contemporaneous pricing data is available showing differences in price between liquid and illiquid shares.
- These studies are commonly relied upon by business valuers because restricted stock studies were one of the few areas where early concentrated research was conducted and actual numerical values were produced. Considerable raw data was available for analysis and many different independent analysts worked the data and produced numerical results.
- Historically, these types of studies were the ones most often accepted by the Tax Court (however, this tendency is being challenged in recent times).

## D. Summary of DLOM Studies/Methods Benchmark

### Weaknesses

- *Lack of Current Market Data*  
The most compelling criticism of existing studies is that they rely on historical market data. A discount for lack of marketability is applied as part of the valuation process to estimate the fair *market* value of an asset or security. With some of the data in the studies reaching back to 1966, it may not reflect the dynamics of current market conditions.
- *Change in Holding Period for Restricted Stocks*  
It is imperative that the expected holding period of the subject company stock be compared to the restricted stock study holding period being used. All except the last two studies use market data pre-April 1997, reflecting the then-current law requiring a two-year holding period prior to sale by an investor of Rule 144 issued restricted stock. The SEC, effective April 1997, amended Section 144 to require only a one-year holding period by investors, implying a lower discount for lack of marketability. The current law, effective February 2008, now requires only a six month holding period by investors of small companies, however no new restricted stock studies have been published, as of yet.
- The studies imply an unusually high return on investment in small company restricted stock.
- *Reliance on averages of restricted stock studies.*  
Using measures of central tendency without an examination of the underlying data leads to the opportunity for mischaracterization of the true restricted stock trading patterns. For example:
  - The Maher Study discount range was 3% - 76%.
  - The Johnson Study range was from a 10% premium to a 60% discount.

The parameters underlying the studies vary by study; some key parameters are listed below:

- 1) Exchange on which the stock trades
- 2) Size of block as a percent of shares outstanding
- 3) Size of company issuing the restrictive shares

## D. Summary of DLOM Studies/Methods Benchmark

### Prevalence in Professional Practice

- Very commonly relied upon in business valuation reports.
- Now seeing trend towards deeper analysis of subject versus the underlying stock in studies—getting behind the data instead of staying at the summary level

### What the Courts say about this Approach:

Courts rejected the use of the **average** restricted stock study results in favor of performing a detailed, comprehensive comparison with underlying restricted stock data.

- *Temple v. U.S.*, No 903-CV-165 (March 10, 2006)  
“The better method is to analyze the data from the restricted stock studies and relate it to the gifted interests in some manner...”
- *Peracchio v. Comm.*, T. C. Memo. 2003-280 (September 25, 2003)  
Paraphrasing: while restricted stock data is helpful in determining a discount for lack of marketability, merely referencing the average discount found in a study or a group of studies, is insufficient.

NOTE: IRS Estate and Gift Tax Program intranet webpage offers summaries of E&G court cases prepared by an IRS Estate Tax Attorney.

Two components to restricted stock study data: a market access component (liquidity), and a holding period component.

- *Holman v. Comm.*, 130 TC 170 (May 27, 2008)  
The Tax Court accepted the expert's use of restricted stock studies in determining DLOM appropriate to gifts of family limited partnership interests. The holding period component deals with the SEC Rule 144 required holding period for a restricted stock sale. Holman concluded that the hypothetical purchaser would demand and get a price concession to reflect the market access component of the marketability discount but would get little if any price concession to reflect the holding period component of that discount.

## D. Summary of DLOM Studies/Methods Benchmark

### FMV Restricted Stock Database—Analysis

The FMV Restricted Stock database of transactions is available for purchase, and is utilized by valuers to estimate DLOM on privately-held business interests. An IRS Engineer completed an analysis<sup>10</sup> of the 475 transactions in the FMV Restricted Stock database in 2009. The purpose was a) to analyze the FMV model for determining DLOM on private equity, and b) to determine whether it is possible to develop a statistically valid regression-based model to determine the DLOM. The conclusions drawn are:

- 1) FMV Opinions' model is flawed insofar as explanation of the DLOM's on the restricted stock transactions in their database;
- 2) Valuers cannot confidently rely on FMV's model when determining DLOM's on restricted stocks, much less on interests in private equity; and
- 3) Neither FMV's model nor multivariate regression analysis can be applied to FMV's database to confidently determine the DLOM on private equity.

FMV Opinions and its principals continue to heavily promote their two-step approach utilizing their database in contributions to various valuation publications and with presentations at various seminars and meetings. Thus, it is likely that we will continue to see this approach used by various practitioners. Before accepting this approach, the reader should familiarize themselves with the analysis and conclusions in Exhibit A and be sure that the result being put forth makes sense in the overall context of the valuation assignment. Please refer to Exhibit A in this job aid for information on the process followed in reaching the conclusions.

### b) Pre-Initial Public Offering (Pre-IPO) Studies

#### Background

The pre-IPO studies are the second large group of studies within the "Benchmark Studies" category. These studies analyze identical stock of the same company and compare price points before the stock is publicly traded and at the point that a liquidity event such as an IPO occurs. Various authors have performed studies using various measuring periods in an attempt to get a stable and reliable pre-IPO stock price for comparison to the price set for the IPO. These measurement points have

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<sup>10</sup> February 18, 2009 memo with the subject, "Update: FMV Opinion's Model and Database". Copy is provided as an Exhibit to this job aid.



#### D. Summary of DLOM Studies/Methods Benchmark

ranged from several days prior to the IPO to several months prior to the IPO. The pre-IPO studies have derived measures of central tendency for DLOM in the area of 30+% to 60+%. Generally, pre-IPO results lead to discount choices higher than those implied by the restricted stock studies.

Traditionally many valuers would consider the results of both the restricted stock studies and the pre-IPO studies, consider the summary statistics and then select a DLOM for use in some subjective matter based on all of these studies. In more recent times, the pre-IPO studies have fallen somewhat from favor due to a significant number of problems identified in their use. The decision in *McCord v. Comm.*, 120 T.C. 358 (2003), pretty much totally rejected the pre-IPO studies as a useful approach to DLOM. A recent court decision, *Bergquist v. Comm.*, 2008 TNT 142-8, has potentially breathed some life back into the pre-IPO studies but this case is a very factually specific case with an extraordinary set of conditions that cannot easily be generalized to other cases.

A pre-IPO study examines arm's-length sale transactions in the stock of a closely held company that has subsequently achieved a successful initial public offering of its stock. In a pre-IPO study, the DLOM is quantified by analyzing (with various adjustments) the difference between the public market price at which a stock was issued at the time of the IPO and the private market price at which a stock was sold prior to the IPO. Three sets of such studies are identified and discussed below:

- Willamette Management Associates
- John Emory
- Valuation Advisors

Studies have shown average discounts of the pre-IPO price from the offering price of around 40% to 45%. Pre-IPO studies have also shown substantial dispersion of the discounts around their sample means

**Willamette Management Associates (WMA):** WMA performed a series of studies on the prices of private stock transactions relative to those of public offerings of stock of the same companies. The studies covered the years 1975 through 1997. See a summary of the studies in Exhibit B. The median discounts ranged from a low of 31.8% for 1991 private transactions to 73.1 % for 1984 private transactions.

## D. Summary of DLOM Studies/Methods Benchmark

**Robert W. Baird & Company Studies (Emory):** John D. Emory of Robert W. Baird & Company conducted another series of pre-IPO studies<sup>11</sup>. The studies covered various time periods from 1981 through 2000. The basic methodology employed in each of the eight studies was identical. The population of companies in each study consisted of initial public offerings during the respective time period in which Baird & Company either participated or for which prospectuses were received. The prospectuses of over 4,000 offerings were analyzed to determine the relationship between (1) the price at which the stock was initially offered to the public and (2) the price at which the latest private transaction occurred up to five months prior to the IPO. The mean discount for all nine studies is 46%. See a summary in Exhibit B.

**Valuation Advisors' Lack of Marketability Discount Study** was developed by Brian Pearson of Valuation Advisors, LLC (VAL), and compares the initial public offering (IPO) stock price to pre-IPO common stock, common stock option and convertible preferred stock prices. These market based transactions demonstrate the lack of marketability discount afforded by the pre-IPO instruments because of their illiquidity when issued by a privately held company.

A summary of Pearson's 1999 Pre-IPO study is available online at [http://www.valuationpros.com/ipo\\_1999.html](http://www.valuationpros.com/ipo_1999.html), the 2000 study at [http://www.valuationpros.com/ipo\\_2000.html](http://www.valuationpros.com/ipo_2000.html) and 2001 at <http://www.valuationpros.com/ipo.html>.

### Areas of Focus

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

- Has Taxpayer's appraiser considered separation of "lack of marketability" from other effects (e.g. management compensation) that might be contributing to discounts observed in the Pre-IPO Studies data?
- Has Taxpayer's appraiser addressed variance and/or range of discounts observed in the Pre-IPO Studies data?

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<sup>11</sup> John D. Emory, "Discounts for Lack of Marketability, Emory Pre-IPO Discount Studies 1980-2000 as Adjusted October 10, 2002", *Business Valuation Review*, Vol.21 No.4 (December, 2002).

## D. Summary of DLOM Studies/Methods Benchmark

- On what basis has Taxpayer's appraiser determined that any particular "average" or "median" discount from the Pre-IPO Studies data is applicable to the subject company?
- On what basis has Taxpayer's appraiser adjusted the average or median discount data for factors applicable to the subject company?
- If Taxpayer's appraiser is using specific pre-IPO transactions from a database, on what basis has Taxpayer's appraiser estimated those particular pre-IPO transactions to be applicable to the subject company?

### Summary

In general, the Pre-IPO studies provide measures of central tendency for DLOM that are higher than those provided by the restricted stock studies. A difficulty in conducting and analyzing these studies is in determining the proper measuring point for the pre-IPO pricing so as not to pick up bias from the market's perception that an IPO or a sale of some other kind is in the wind. One must be cautious as to going too far back, however, because market conditions in general and for the company in specific could have changed markedly over time, especially if the company is small and in a highly competitive industry.

### Strengths

- Empirical evidence, market data
- Broad time period coverage

### Weaknesses

- Not contemporaneous – too much time gap often exists between pre-IPO transaction and public offering. Private transactions studied were between 5 months and 3 years prior to the IPO, providing a strong argument that factors other than marketability alone led to the price increase.
- Pre-IPO companies rapidly evolving – significant changes (difference in pre & post company) as many transactions involved companies in early stage of development
- Pre-IPO companies affected by changes in economic conditions
- Data includes only firms with successfully completed IPO's. No information included on candidate companies where IPO doesn't eventually take place
- Pre-IPO transactions tend to be under-priced (most IPOs involve high growth companies) to fully subscribe the offering

## D. Summary of DLOM Studies/Methods Benchmark

- Pre-IPO transactions almost always involve related-party transactions (employees and company, service providers and company, etc) and do not reflect arms-length terms.
- There are indications that the *Willamette Management Associates Studies* 1999 and 2000 data may be skewed due to the dot.com "bubble"
- Frequently viewed as inflating DLOM

### Important parameters for this approach

- 1) Price stock initially offered to the public
- 2) Price at which latest presumably unaffected, private transaction occurred prior to IPO (time period varies by study)

### Prevalence in professional practice

Not as common in practice as Restricted Stock studies after McCord case where pre-IPO studies were rejected; decision in Bergquist could bring new life.

### What the Courts say about this approach

There have been numerous court decisions where the Pre-IPO approach to DLOM was considered. Among these are the following:

- *Estate of Gallo* (T.C. Memo 1985-363, 50 T.C.M. (CCH) 470
- *Estate of Hall* 92 T.C. 312 (1989)
- *Howard v. Shay* (1993 U.S. Dist. LEXIS 20153 (C.D. Cal.1993), rev'd and remanded, 100 F.3d 1483 (9<sup>th</sup> Cir. 1996), cert. denied, 520 U.S. 1237 (1997)
- *Okerlund v. United States* (53 Fed. Cl. 341 (Fed. Cr. 2002), motion for new trial Denied, 2003 U.S. Claims LEXIS 42 (Fed. Cl. 2003)
- *McCord V. Commissioner* 120 T.C. No. 13 (2003) decision effectively disavowed the pre-IPO studies approach
  - Rejected the use of pre-IPO studies to determine the appropriate discount
  - En banc decision – entire Tax court examines controversy
  - No dissent regarding rejection

## D. Summary of DLOM Studies/Methods Benchmark

- *Bergquist v. Commissioner* 131 T.C. No. 2 ( 2008)
  - 2008 charitable contribution case allowed expert's report that relied on the Pre-IPO Approach without discussion of the approach.
  - According to the Court, the taxpayer's expert "[has] not pointed to, nor do we find, significant flaws in respondent's expert's analysis or in the studies he relied upon that would suggest his report is unreliable, and we adopt [the IRS's expert's] discounts and conclusions of value."
  - It is critical that facts be developed and valuation is based on specific facts for the subject company.

### c) Restricted Stock Equivalent Analysis<sup>12</sup>

This approach is a recent attempt to refine the traditional restricted stock studies approach to consider the real differences existing between the marketability of the restricted stock of publicly traded companies and the stock of companies that are not publicly traded and that, therefore, do not have only a limited period of lack of marketability. It derives a proposed DLOM as a two step process starting with the so-called "Restricted Stock Equivalent DLOM".

- 1) Estimate the "Restricted Stock Equivalent Value" for application to the publicly traded stock
- 2) Add an increment to the restricted stock equivalent value to account for difference in marketability of the restricted stock of public companies versus the subject private stock

This approach to DLOM is fully described in a number of papers authored by Espen Robak and Lance Hall of FMV Opinions.

The essence of this approach is that straight restricted stock analysis misses the true characterization of DLOM for private companies because it relies totally on data relating to public companies, even though it focuses on the restricted stock of those companies. Per its supporters, private companies are even less marketable than the restricted stock of public companies and thus an extra increment of discount is appropriate. The proponents of this approach quantify this increment using data collected

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<sup>12</sup> Espen Robak, "Liquidity and Levels of Value: A New Theoretical Framework," BV Update, October, 2004.

## D. Summary of DLOM Studies/Methods Benchmark

on a substantial number of restricted stock transactions by using the discount difference between the largest block sizes of purchased restricted stock and the smallest block sizes of such stock as an indicator of the additional marketing risk faced by private companies.

Conceptually the approach proceeds as follows:

- 1) To get the restricted stock equivalent value, select restricted stocks with characteristics as close to your subject as possible in terms of risk and distributions
- 2) Proxy for risk includes:
  - **Size** (as measured by assets, resources, or estimated pre-discount market value of equity)
  - **Profitability** (as measured by dollar amount of some level of profitability or percent of profitability)
  - **Balance sheet risk** (as measured by some measure of leverage or pre-discount estimated market value of equity to book value of equity)
- 3) Distributions are usually measured as the proportion of dividends or withdrawals paid out as a percentage of the pre-discounted market value of equity
- 4) Estimating the Private Company Incremental Discount
  - Large blocks of restricted stock relative to total shares outstanding are much closer to private equity than the typical smaller block of restricted stock
    - **Silber Study**<sup>13</sup> of restricted stock
    - Fewer prospects in the pool of potential buyers
    - Longer period to feed out into public market under the SEC dribble out rule
- 5) Difference between the average discount on the sample of small block restricted stocks with the characteristics similar to the subject and the average discount for a large block of stock would be the private stock liquidity increment to the discount

This approach is treated in more detail in Exhibit A to this job aid including a statistical analysis by an IRS Engineer<sup>14</sup>. The conclusions drawn are:

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<sup>13</sup> William L. Silber, "Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices," *Financial Analysts Journal*, July-August 1991, pp. 60-64.

<sup>14</sup> February 18, 2009 memo with the subject "Update: FMV Opinions' Model and Database".

## D. Summary of DLOM Studies/Methods Benchmark

- 1) FMV Opinions' model is flawed insofar as explanation of the DLOM's on the restricted stock transactions in their database;
- 2) Valuers cannot confidently rely on FMV's model when determining DLOM's on restricted stocks, much less on interests in private equity; and
- 3) Neither FMV's model nor multivariate regression analysis can be applied to FMV's database to confidently determine the DLOM on private equity.

FMV Opinions and its principals continue to heavily promote their two-step approach utilizing their database in contributions to various valuation publications and with presentations at various seminars and meetings. Thus, it is likely that we will continue to see this approach used by various practitioners. Before accepting this approach, the reader should familiarize themselves with the analysis and conclusions and be sure that the result being put forth makes sense in the overall context of the valuation assignment.

The approach is relatively new and has not had any significant vetting in the practitioner community or by the courts.

### Areas of Focus

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

- Has Taxpayer's appraiser considered separation of "lack of marketability" from other effects (e.g. blockage) that might be contributing to discounts observed in the Restricted Stock Equivalent data?
- Has Taxpayer's appraiser addressed variance and/or range of discounts observed in the Restricted Stock Equivalent data?
- On what basis has Taxpayer's appraiser determined that any particular "average" or "median" discount from the Restricted Stock Equivalent data is applicable to the subject company?
- On what basis has Taxpayer's appraiser adjusted the average or median discount data for factors applicable to the subject company?
- If Taxpayer's appraiser is using specific restricted stock equivalent transactions from a database, on what basis has Taxpayer's appraiser estimated those particular restricted stock transactions to be applicable to the subject company?

## D. Summary of DLOM Studies/Methods Benchmark

### d) Cost of Flotation<sup>15</sup>

The flotation cost approach quantifies the discount for lack of marketability in terms of the costs required to achieve marketability. The DLOM is thus the cost to underwrite a public offering of the stock as a percentage of the estimated traded price that would result from such an offering. Certain observations on this approach follow:

- Costs include the legal, accounting, and investment banking fees necessary to underwrite and place an issue with investors and typically includes a high degree of due diligence
- 1972 SEC study indicated flotation costs of 21.2% for 270 stock issues up to \$1 million and 12.2% for 1,008 stock issues of \$1 to \$10 million
- Not applicable to minority interests since cannot cause a public offering
- A second way to estimate illiquidity cost for controlling interest is to look at expense of selling the business
- Deemed to be only a portion of the DLOM
- Does not reflect the risk associated with uncertain holding period for a non-marketable investment

The approach is easily applied and a wealth of data is available. However, it does not reflect the risk associated with the uncertain holding periods that are typical for an illiquid investment in private equity and therefore, does not quantify the entire DLOM. It is also not applicable to minority interests which are the most frequent interests in question when a discount for lack of marketability is to be estimated.

### Areas of Focus

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

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<sup>15</sup> Cost of Floatation of Registered Issues 1971-1972, Washington, DC: Securities and Exchange Commission, 1974. See also The Costs of Going Public, Jay R. Ritter, *Journal of Financial Economics*, Vol. 19, No. 2 (December 1987), pp. 269-81.



## D. Summary of DLOM Studies/Methods Benchmark

- Has Taxpayer's appraiser considered separation of "costs of reaching marketability" from other effects (e.g. other expenses) that might be contributing to discounts observed in the Flotation Costs data?
- Has Taxpayer's appraiser addressed variance and/or range of discounts observed in the Flotation Costs data?
- On what basis has Taxpayer's appraiser determined that any particular "average" or "median" discount from the Flotation Costs data is applicable to the subject company?
- On what basis has Taxpayer's appraiser adjusted the average or median discount data for factors applicable to the subject company?
- If Taxpayer's appraiser is using specific flotation costs transactions from a database, on what basis has Taxpayer's appraiser estimated those particular flotation cost transactions to be applicable to the subject company?

### e) Mandelbaum Factors, Judge Laro, 1995

The Mandelbaum Factors were set out in a Tax Court case<sup>16</sup> of the same name decided by Judge Laro as an approach to adjusting the discount for lack of marketability achieved by traditional means such as the Benchmark Studies for the specific facts and circumstances of the valuation problem actually being considered. The factors and the analysis that go with them have since been cited in several following court decisions and are considered by many valuers to form a good conceptual basis for thinking about and quantifying DLOM. The courts have emphasized, however, the process defined in Mandelbaum as opposed to the actual quantitative result that was achieved in that case.

Summary:

Per Judge Laro, the following factors should be addressed as they pertain to a discount for lack of marketability for the subject company.

#### **1. Private vs. public sales of stock**

In the event that a company has observable transactions between third parties that involve both their publicly traded stock and restricted shares, this point has important application. If the subject shares do not have a publicly traded counterpart, review of the restricted stock studies can serve as an important reference.

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<sup>16</sup> *Mandelbaum v. Commissioner*, T.C. Memo 1995-255 (1995)

## D. Summary of DLOM Studies/Methods Benchmark

### **2. Financial Statement Analysis**

Financial statement analysis would include historical and projected trends in profitability, leverage, distributions, liquidity, and volatility of these and other measures.

### **3. Company's Dividend Policy**

Investors in non-marketability securities prefer distributions as they provide elements of capital recovery and capital gain

### **4. Nature of the Co. (History, Position in Industry, Economic Outlook)**

Investors gravitate to positive results and shy away from risk

### **5. Company's Management**

Intangibles such as management contribute to operational and financial success and help to ensure favorable returns

### **6. Amount of Control in Transferred Shares**

Control or related influence will typically be perceived as reducing risk

### **7. Restrictions on Transferability of Stock**

Specific clauses that are viewed as unattractive and tend to increase discounts:

- a) Right of First Refusal – many limited partnership agreements provided that a limited partner can only sell and transfer an interest subject to first offering to sell that interest to the partnership or its partners. These provisions are onerous as they impair an interest's marketability by discouraging third party offers.
- b) Transferee Restrictions – such as limits on transfer to "permitted transferees" reduce the universe of potential buyers and generally lengthen the time horizon to liquidate the investment

### **8. Holding Period for Stock**

The key is whether such holding period is discretionary or mandated. Restrictions on holding are clearly perceived as negative by investors. But for non-marketable securities, the loss of vital timing in being able to liquidate an investment can be regarded as a substantial negative to a prospective investor that is faced with an uncertain time horizon and outlook, including impacts of overall capital markets

### **9. Company's Redemption Policy**

Put rights or expectations of near term monetization events reduce the risk to an investor

## D. Summary of DLOM Studies/Methods Benchmark

### **10. Costs Associated with Making a Public Offering**

While public offerings are under the control of the corporation or majority owner, these provisions only related to marketability. Even marketable securities can be impacted by severe liquidity discounts during bear markets,

#### Areas of Focus

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

- Has Taxpayer's appraiser considered each of the Mandelbaum Factors in the estimation of the discount for lack of marketability?
- On what basis has Taxpayer's appraiser determined the relative importance of each of the Mandelbaum Factors towards the estimation of the discount for lack of marketability?
- On what basis has Taxpayer's appraiser adjusted the average or median discount data for effects from each of the Mandelbaum Factors?

#### Strengths

- Raises importance of the skilled application of difference/similarities of benchmark studies to subject company
- Similarity to precepts underlying Rev. Ruling 59-60, 1959-1 CB 237

#### Weaknesses

- Attempt to cover all ten Mandelbaum factors might be difficult unless experienced
- Insufficient information to analyze and provide opinion on all factors

#### Prevalence in professional practice

- Increasingly common; how factors are applied and the magnitude of the effect on marketability discount is problem

#### What the Courts say about this Approach

The Mandelbaum approach has received a considerable amount of attention among business valuation practitioners and in the courts. Among the lessons learned are that:

#### D. Summary of DLOM Studies/Methods Benchmark

- (1) Detailed data developed first hand by the testifying expert, as opposed to medians cited from studies performed by others, are required to sustain discount opinions
- (2) The courts recognize there are reasons to go above or below the medians, but they will do so only when presented with soundly reasoned and empirically supported arguments
- (3) One size discount should not apply to all
- (4) Blanket approaches using historical averages are not sustainable; a case-specific analysis is needed

For example, in the *Estate of Jelke v. Commissioner*, T.C. Memo 2005-131, reversed and remanded, 507 F.3<sup>rd</sup> 1317 (11<sup>th</sup> Cir. 2007), cert. den. 129 S. Ct. 168 (2008), the court said that they found the factors considered in Mandelbaum to be a helpful guide to determining the marketability discount and in structuring their own Mandelbaum-type analysis. Thus, the court followed a Mandelbaum process but did not blindly endorse the original Mandelbaum result.

## 2. Securities-Based Approaches

The security-based approaches to estimating the Discount for Lack of Marketability are based on theoretical option pricing models (e.g. Longstaff, Chaffee) and from observing illiquidity demonstrated by traded stock prices (bid-ask-spread) and option prices (LEAPS).

### (a) Long-Term Equity Anticipation Securities (“LEAPS”) – Robert Trout, 2003, and Ronald Seaman, 2005

#### Background

Long-Term Equity Anticipation Securities or (“LEAPS”), which are publicly traded, are long-term put options on stocks of public companies.

Robert Trout originally published the LEAPS study in September 2003<sup>17</sup> and Ronald Seaman updated the study in June 2005<sup>18</sup>, September 2007<sup>19</sup> and March 2009<sup>20</sup>. A LEAP is a long-term put option with a term of approximately 1.5 to 2.0 years. An investor, therefore, can buy protection against stock price declines by purchasing a LEAP put option. The LEAP studies examined the cost of purchasing the LEAP puts. The DLOM is then calculated as the cost of the put option divided by the stock price. The authors segmented the data by a safety rank measured by the Value Line Investment Survey with 1 representing the least risk and 5 representing the most risk.

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<sup>17</sup> Robert R. Trout, “Minimum Marketability Discounts,” *Business Valuation Review*, September 2003 pp. 124-126.

<sup>18</sup> Ronald M. Seaman, “Minimum Marketability Discounts 2<sup>nd</sup> Edition,” *Business Valuation Review*, June 2005 pp. 58-64.

<sup>19</sup> Ronald M. Seaman, “Minimum Marketability Discounts—3<sup>rd</sup> Edition,” September 2007, available at <http://www.dlom-info.com/>.

<sup>20</sup> Ronald M. Seaman, “Minimum Marketability Discounts—4th Edition, A Study of Discount for Lack of Marketability Based on LEAPS Put Options in November 2008,” March 2009, available at <http://www.dlom-info.com/>.

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

### Summary:

The authors concluded that the observed DLOM derived from the LEAPS studies should be viewed as a benchmark minimum price when applied to privately held companies. They viewed the derived discounts as minimum price discounts since

- a. The market value of the companies offering the underlying securities was much larger than the value of a privately held company
- b. The underlying securities are marketable
- c. The LEAPS can be sold at any time during the holding period
- d. There is a known liquidity event for LEAPS e.g. the option has an expiration date generally between 1.5 and 2.0 years.

One year median discounts ranged from 8.3% for the safest company to 17% for the riskiest company. Two year median discounts ranged from 9.3% for the safest company to 31% for the riskiest company. A one year or two year implied discount would be used as a proxy depending on the length of time it would take to market the subject interest (e.g. for a controlling interest a one-year discount would generally be used as a proxy since it is easier to market a controlling interest in a privately held concern than it is to market a minority interest).<sup>21</sup>

One area in which there has been criticism of using the LEAPS data as a starting point for the DLOM is that it only considers the cost of purchasing a put option, which protects an investor from a downward price movement. Therefore only the cost to purchase a put option is considered if using LEAPS data to develop a DLOM. If an investor can purchase a put (Protective Put) to protect against a downward movement in the stock the investor can also sell a call option (Covered Call) and receive a premium to offset the cost of the put. As a result the overall cost is reduced since the investor is receiving a premium for selling the call. Purchasing a put option and selling a call on the underlying stock is called a “collar” options strategy.<sup>22</sup>

At issue here is whether an investor in a privately held company, if they had the ability to hedge, would only purchase a put to protect against a price decline or purchase a put and sell a call locking in the current price and foregoing unlimited future profit potential<sup>23</sup>

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<sup>21</sup> For additional reading refer to BVR’s 2008 Guide to Discounts for Lack of Marketability –The Use of Theoretical Models to Estimate the Discount for Lack of Marketability, by Travis R. Lance.

<sup>22</sup> For additional information on the collar options strategy refer to <http://www.optionseducation.org/strategy/collar.jsp>

<sup>23</sup> See <http://www.dlom-info.com/q-and-a.html> for response to collar options strategy by Seeman

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

### Strengths

- There are more than twice the number of LEAPS transactions in the LEAPS study than are considered in the restricted stock studies thereby providing a more valid statistical sample
- LEAPS can be found that are valuation date specific
- Data can be segmented by industry and a search can be conducted for comparable public companies

### Weaknesses

- An appraiser would still have to perform a qualitative analysis in order to arrive at a conclusion for the DLOM by adjusting the LEAPS based discount for private company considerations
- An owner of a privately held company does not have the ability to hedge the investment in an options market and as such the observed discount is a proxy and other qualitative factors must be considered to arrive at a final conclusion

### Important Parameters for this Approach:

- Based on market data for the price and Value Line Investment Survey reports to assess the safety factor

### Prevalence in Professional Practice:

Not seen very often, particularly for closely held companies.

### What the Courts say about this Approach (include cite):

This approach has not been vetted in any meaningful way by the courts.

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

### (b) The Longstaff Study, Journal of Finance, December 1995<sup>24</sup>

#### Background

Francis A. Longstaff authored a study that relies on stock option pricing theory to estimate the DLOM for a privately held company. The Longstaff study is based on the price of a “look back” option<sup>25</sup>. Using option-pricing theory the model relies on the restriction period and the volatility or standard deviation of a security’s return. Essentially Longstaff assumed that an investor with perfect timing ability would have the ability to identify a point in time in which the security price reaches its maximum value. If an investor is locked up for a certain period of time the investor gives up the opportunity to sell the security at its maximum price. The marketability discount in Longstaff’s model is the present value difference between what the investor could sell the security for after the marketability restrictions have lapsed and the maximum price the security could have sold for during the restriction period.

One of Longstaff’s observations is that the discount for lack of marketability can be economically significant even with a very short restriction period as can be seen in the sample output below.

Sample Outputs from Longstaff Model			
Term	Volatility of Underlying Stock		
	10%	20%	30%
1 Day	0.40%	0.80%	1.30%
30 Days	2.30%	4.70%	7.10%
180 Days	5.80%	11.80%	18.10%
1 Year	8.20%	17.00%	26.30%
5 Years	19.10%	41.0%	65.80%

It should be noted that the above table is for illustrative purposes only and the data points above were referenced in the author’s study. Volatilities in excess of

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<sup>24</sup>Longstaff, Francis A., “How Much Can Marketability Affect Security Values?”, *The Journal of Finance*, Vol. L, No. 5 (1995), pp.1767-1774.

<sup>25</sup>A **look back option** is a path dependent option that is settled based upon the maximum or minimum underlier value achieved during the entire life of the option. Essentially, at expiration, the holder can “look back” over the life of the option and exercise at a value based upon the optimal underlier value achieved during that period. Look backs can be structured as puts or calls and come in two basic forms: A fixed strike and a floating strike.



#### D. Summary of DLOM Studies/Methods Securities-Based Approaches

30% would most likely be used as a proxy. Therefore, the model may produce results which are not realistic as indicated in the table below.

Longstaff Model Discounts as a function of Time and Volatility							
Term	Volatility						
	10%	20%	30%	40%	50%	60%	70%
1 Day	0.40%	0.80%	1.30%	1.70%	2.10%	2.50%	3.00%
30 Days	2.30%	4.70%	7.00%	9.50%	12.00%	14.50%	17.00%
180 days	5.70%	11.70%	18.00%	24.50%	31.20%	38.30%	45.70%
1 Year	8.20%	17.00%	26.30%	36.10%	46.60%	57.60%	69.20%
2 Years	11.80%	24.60%	38.60%	53.70%	70.10%	87.70%	106.70%
5 Years	19.10%	41.00%	65.80%	93.70%	125.00%	159.90%	198.50%

#### Summary

The Longstaff approach assumes perfect market timing and, therefore, derives an upper bound for the lack of marketability discount since an investor is looking backward in time to make his buy/sell decisions instead of making these decisions based on present evidence and anticipated future stock price movements. Volatilities in excess of 30% would most likely be used as a proxy for privately held stock for which there is no public market. Therefore, the model may produce results which are not realistic as the upper bound in circumstances of this kind could well reach 100%.

#### Strengths

- The model can be easily implemented in Excel and provides a benchmark maximum estimate on the discount for lack of marketability.

#### Weaknesses

- The Longstaff model assumes that the investor has perfect market timing and that the investor has trading restrictions that prevent the security from being sold at an optimal time. Absent these restrictions, the investor would know the exact best time to exercise the option and sell the underlying stock and would do so.
- The Longstaff model produces very high marketability discounts with relatively low volatility of 30%. Most small cap companies have volatilities in excess of 50%. The model produces an estimate of an "upper boundary" for DLOM.
- As mentioned previously the model should be used as a proxy for a maximum estimate and should not be used blindly to determine a discount for lack of marketability

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

Important Parameters for this Approach:

- Time to expiration
- Volatility

Prevalence in Professional Practice:

The model is not seen often for estimating DLOM for a privately held company. It is more useful for estimating the discount on a large block of restricted stock of a publicly traded company.

What the Courts say about this Approach:

This approach has not been vetted in any meaningful way by the courts.

### (c) The Chaffee Study

#### Background

In 1993 David B.H. Chaffee authored a study, which related the cost to purchase a European put option to the measurement of the Discount for Lack of Marketability.

#### Summary

In 1993 David Chaffee III published an article on his theory that the Black Scholes Option Pricing Model could be used to determine the DLOM. He found that the European Option<sup>26</sup> exercisable only at expiration was an appropriate model for the SEC Rule 144 Holding Period of restricted shares.<sup>27</sup>

Chaffee relied on the Black Scholes Option Pricing Model for a put option to determine the cost or price of the put option. The cost of the put option divided by the market price equals the Discount for Lack of Marketability ("DLOM").

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<sup>26</sup> European options can be exercised only at maturity. American options can be exercised early.

<sup>27</sup> See James R. Hitchner, Financial Valuation, 2<sup>nd</sup> Edition John R. Wiley and Sons 2006.

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

Chaffee determined his proxy of a Discount for Lack of Marketability based on volatilities in excess of 60% based on analysis of small Over the Counter ("OTC") public companies.

The appropriate DLOM for a stock with a two-year holding period and a volatility between 60% to 90% according to Chaffee was between 28% and 41% which he cited as similar to the restricted stock studies.

Chaffee increased the holding period to 4 years, which showed a range of DLOM from 32% to 49%. Increasing the holding period to greater than four years did not materially change the DLOM.

According to Chaffee the use of the Black Scholes Option Pricing model for European options produced a minimum DLOM since a European put option pricing formula does not take into account early exercise.

### Strengths

- The model can be easily implemented in Excel and is based on the European put option Black Scholes Formula

### Weaknesses:

- As with all the option pricing models mentioned in this section, the DLOM should be used as a proxy only. Other qualitative factors must be considered to determine a final DLOM.
- Chaffee considered his results as "downward" biased and as such his findings are considered a minimum DLOM
- The owner of privately held company stock does not have the ability to hedge their investment. The option models provide a proxy for marketability and the model can't be used without consideration to other factors.

### Important Parameters for this Approach

- Stock price and exercise price are equal. The stock price and exercise price is equal to the marketable value of the privately held stock at the Valuation Date

#### D. Summary of DLOM Studies/Methods Securities-Based Approaches

- The rate is equal to the Weighted Average Cost of Capital
- Volatility is based on comparable publicly traded guideline companies
- The term is equal to the length of time the security is expected to remain non-marketable

#### Prevalence in Professional Practice

Not seen very often, particularly in valuations of private companies.

#### What the Courts say about this Approach

This approach has not been vetted in any meaningful way by the courts.

#### (d) Bid-Ask Spread Method to Determine DLOM<sup>28</sup>

##### Background

The bid-ask spread is the difference between the price asked for the business by the seller ("ask price") and the price offered for the business by the buyer ("bid price"). The illiquidity is measured as the percentage difference between the bid and the ask price. In most markets, there is a dealer or market maker who sets the bid-ask spread to cover its costs of holding inventory, processing orders and trading with more informed investors. The spread has to be large enough for the dealer to cover his costs and yield a reasonable profit. Amihud and Mendelson tested market rates of return against yield spreads (difference between bid-ask price) for various financial stocks for the period 1961-1980. Their regression was significant. This signifies that the returns on the stocks were not only a function of risk but also of illiquidity. Therefore, the riskier the stock, the larger the spread and the higher the implied DLOM<sup>29</sup>.

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<sup>28</sup> Amihud, Yakov, and Mendelson, Haim. "Asset Pricing and the Bid-Ask Spread," *J. Financial Econ.* v 17 (December 1986): 223-49.

<sup>29</sup> See Kasper Larry J, *Business Valuations: Advanced Topics*, Greenwood Publishing Group 1997 Chapter 5 Premium and Discounts

## D. Summary of DLOM Studies/Methods Securities-Based Approaches

### Summary

This is a conceptually simple approach and utilizes actual market data. Market makers are market savvy and could be inclined to over-estimate the implied DLOM to build a spread that will bring them increased profits. The more traders that there are in the marketplace the better the bid-ask spread should represent the actual effects of lack of marketability.

### Strengths

- There is a large sample of trading firms from which an illiquidity discount can be computed

### Weaknesses

- This approach provides an illiquidity discount only. Other factors such as restrictions on marketability need to be considered to get to DLOM
- The bid-ask spreads of publicly traded stocks must be related to variables that can be measured for a private business
- Considerable subjective judgment is still required on the part of the valuator

### Important Parameters for this Approach:

- In order to apply this approach to a private company, a model could be developed which could, for example, take into consideration such parameters as revenues, a measure of size, and whether a firm was profitable or not.

### Prevalence in Professional Practice:

This approach is not seen very often for estimating DLOM for a privately held company.

### What the Courts say about this Approach

This approach has not been vetted in any meaningful way by the courts.

### **3. Analytical Approaches**

#### **Approach Overview**

The analytical approach studies the discount for lack of marketability (DLOM) through the consideration of various transactional data sets. The involved data sets have been put together by the authors of the DLOM studies from various sources and number from less than 100 to several hundred sale transactions involving stock sales conducted outside the public market place. The sales normally involve the stock issuer as seller and various institutional entities as buyers thus by-passing the normal registration requirements of the U. S. Securities and Exchange Commission (SEC) for stock to be sold to the general public. These data sets generally compare the sale price for blocks of publicly traded stocks sold through private placements as compared to the sale price of the shares as traded on the primary market where such are listed. These data sets are analyzed statistically and through regression analyses to both determine the total amount of the discount and the breakdown of that discount across various postulated causal factors. The types of data in question are similar to those that form the basis of the better known “restricted stock studies” that are the subject of another portion of this DLOM job aid.

The transactions that make up the dataset are screened in various ways to eliminate outliers and to identify any specific factors relating to the private placement that are not comparable to the factors that are attributable to associated traded shares that also constitute minority interests. A typical private placement block size might be 15% of the total outstanding common stock. Where significant size blocks are involved in comparison to normal daily trading volumes for the associated stock on the public marketplace, some aspect of blockage discount as well as regular DLOM may be present in the transaction. The valuation analyst needs to be alert to such a possibility.

There have been a number of different researchers starting in the late 1980's that have taken an analytical approach to estimating DLOM. Almost all of these researchers come from the academic community and none started out his or her research with tax concerns in mind. The research was undertaken for various purposes but the fundamental underlying intent was to better understand the characteristics of capital formation among public companies. Typical questions posed for study are when should debt be issued instead of stock, when should preferred equity be issued instead of common equity, when should common

## D. Summary of DLOM Studies/Methods Analytical Approaches

equity be issued instead of debt or preferred equity and what mode of issuance should be used.

Four such studies are reviewed below. The first two are early studies (Wruck – 1989 and Hertz & Smith – 1993) that illustrate the methodology for an event-based analysis often used in corporate capital formation investigations. The third study is the Bajaj et al analysis that has been referred to in a number of court cases that have been tried since the year 2000 and the fourth study is a portion of a body of work by Ashok Abbott that has drawn recent attention to the area of discounts for lack of liquidity (DLOL). Lack of marketability (LOM) and lack of liquidity (LOL) have often been treated in the literature as identical concepts. However, these two areas have been distinguished by Pratt<sup>30</sup> and Abbott<sup>31</sup> as follows. Per Pratt, marketability is the legal ability to sell an asset whereas liquidity is the ability to sell an asset without delay and without loss of value. Per Abbott, marketability denotes the right to sell an asset in an established and efficient public capital market, within a reasonable time, with relatively low transaction costs, and with minimal effect on that security's public market price. Liquidity, on the other hand, denotes the ability to convert an asset into cash without diminishing its value.

In addition to these studies, Exhibit C – Analytical Approach Revisited provides a summary of several additional studies that further expand the analysis of the mechanics of corporate capital formation.

### Terminology

The analytical studies are usually configured as “event studies” which involve the “private placement” of “unregistered” or “registered” shares of stock or, in some cases, both. These terms are explained below.

**Event Study** – a study that investigates the circumstances surrounding and the results of a specific event such as a private placement of stock in bulk.

**Private Placement** – a transaction involving a seller which is usually the issuer of a class of stock and a buyer which is usually a large investor such as an institution or a stock fund outside of the normal market mechanism of a public stock exchange. Evidence indicates that the per share price of a private placement transaction is often set at a discount to the publicly traded price of the same stock as quoted in the market.

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<sup>30</sup> Pratt, Shannon P., *Business Valuation Discounts and Premiums*, Wiley 2001, pg 10

<sup>31</sup> Abbott, Ashok B., Presentation at Business Valuation Conference: Summit on Lack of Marketability, University of San Diego School of Law, September 18, 2008, Slide 4

## D. Summary of DLOM Studies/Methods Analytical Approaches

Unregistered Share – a share of stock that has not been registered under the Securities and Exchange Act of 1933 and thus which cannot be sold to just any interested investor or traded on the public exchange. Such shares can be sold to specific sophisticated investors such as those noted above.

Registered Share – a share of stock that has been registered under the Securities and Exchange Act of 1933 and that can be freely sold to any investor desiring to buy.

In many cases, both registered and unregistered shares sell at a discount when privately placed in bulk. If one assumes that a registered share is freely tradable to anyone at any time then a marketability discount of zero would pertain to that share. Thus, by comparing the total discount per share for the private placement of unregistered shares to that of registered shares, analysts can obtain an estimate of the discount for lack of marketability for those shares since all other discount factors should be the same.

Some analysts dispute this approach on the grounds that even registered shares do not necessarily have a lack of marketability discount of zero if such are offered in bulk or are thinly traded in the marketplace. Under this logic the difference between the price of unregistered shares and registered shares offered in private placements would tend to under-estimate the discount for lack of marketability. For example, if the difference in total discount is 10% but the registered shares already have a 5% discount for DLOM built-in then the actual DLOM for the unregistered shares is 15% rather than 10%. As the unregistered shares serve as a surrogate for the shares of a non-publicly traded entity the substance or lack thereof in the question raised above can be of some importance (in the illustration this amounts to an increase in DLOM of 50%).

### Primary Reviewed Studies

#### (a) Karen Hopper Wruck<sup>32</sup>

##### Background and Summary

Karen Wruck studied equity ownership concentration and its effects on firm value. Her premise was that private placements act to increase ownership

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<sup>32</sup> Wruck, Karen Hopper, "Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings", *Journal of Financial Economics* 23 (1989), 3-28.



## D. Summary of DLOM Studies/Methods Analytical Approaches

concentration by bringing aboard more large shareholders and that such increased concentration should manifest itself in an overall increase in firm value thereby benefiting all shareholders. On the other hand, public offerings of equity tend to dilute share value for the existing ownership base.

Wruck studied 128 private sales of equity involving 65 companies traded on the New York Stock Exchange and 63 companies traded on the American Stock Exchange between July 1979 and December 1985. She measured the share price of the stock on the exchanges 1 day after the announcement of the private placement and compared that price to the share price involved in the placement. She considered both placements of registered shares and placements of unregistered shares and found a 17.6% average difference in discounts between the two types of shares when privately placed. The median difference in discounts was 10.4%.

Wruck concluded that private placements of all types sell at a discount to the publicly traded shares but that unregistered shares required a higher discount for placement than registered shares. It was postulated that the need for this higher discount was a function of lack of marketability as well as the increased costs of monitoring borne by investors that hold unregistered shares. She hypothesized that private placements are generally bought by active investors that act to keep management on its toes thereby positively affecting overall firm value.

Since monitoring costs are involved to some extent for private placement investors, whether their shares are registered or unregistered, users of the Wruck study have postulated that the 17.6% average discount difference is primarily related to lack of marketability for the unregistered shares. However, since the Wruck analysis did not control for the effects of other potential contributing variables, it is quite possible that a meaningful portion of the average discount difference could be caused by existing operational differences in data set firms rather than to marketability.

### Strengths

- The strength of the Wruck analysis is its clearly defined hypothesis and the use of analytical tools to investigate that hypothesis. Both registered and unregistered placements are considered with companies listed on two different exchanges represented almost equally.
- The discount result reached is logically supported by the analysis approach used.
- Further, by comparing registered and unregistered placement discounts, the Wruck methodology presents a way of isolating the discount for lack of

## D. Summary of DLOM Studies/Methods Analytical Approaches

marketability from certain other factors such as assessment and monitoring costs that could also lead to discounts.

### Weaknesses

- The weakness of the Wruck analysis relates to the data selection approach that was utilized. The sample of firms chosen seems to have been primarily based on data availability rather than logical selection methodologies.
- The classification determination as to whether placed shares were registered or unregistered was predicated on published reports in the Wall Street Journal. Of the 128 firms in the sample, a determination was available for only 73 of the placements and that determination was subjective in nature.
- The measurement point was chosen as one day after the announcement date which would seem to take advantage of any immediate bounce in stock price thereby increasing measured discount amounts. This weakness is somewhat mitigated by the methodology that compares registered discounts to unregistered discounts instead of measuring unregistered discounts in total as an indication of lack of marketability. Thus, assuming that a market bounce might result from any private placement as Wruck hypothesizes, the difference in discount existing between the two types of placements might still be a valid measure of lack of marketability effects.

### Important Parameters of the Study

The primary parameters in the Wruck study are:

- the selection of the sample itself,
- the registration status of the placement, and
- the selection of a proper measurement point.

### View of the Valuation Community

The Wruck study has been cited by a number of practitioners but is basically utilized as background material to introduce the subject of investigating marketability discounts analytically. Although the average and median discounts of the study are offered as evidence that the results of the benchmark studies may be much too high, the Wruck discounts themselves are not offered as actual discount proposals.

## D. Summary of DLOM Studies/Methods Analytical Approaches

### View of the Courts

Since the numerical values of the Wruck discounts have not been advanced in court as actual discount amounts, the courts have not specifically opined on the Wruck study and its results.

### (b). Hertz and Smith<sup>33</sup>

#### Background and Summary

Hertz & Smith (H & S) studied market discounts and shareholder gains involved in the private placement of equity. They hypothesized that private equity placements are often undertaken by firms with limited tangible assets, by firms engaged in the speculative development of new products and by firms in financial distress. Due to the higher risk inherent in these types of firms, they tend to offer private placements priced at higher than normal discounts. These higher than normal discounts compensate investors for the higher information costs incurred and the higher monitoring costs required to keep suitably informed of investment status. Based on these premises, H & S believed that the discounts required to sell equity privately existed for a number of reasons beyond the potential lack of marketability of the purchased shares or the expectation that the buyers would provide services as well as investment capital.

H & S used statistical analysis techniques to identify those factors that contribute to the overall observed discount; including, but not limited to, the effects of lack of marketability. H & S found an average discount differential between private placements of unregistered shares as compared to private placements of registered shares of 13.5%. They considered this to be a surrogate for DLOM. However, they opined that this surrogate should not be accepted on face because they believed that if the DLOM discount was really this high, then firms would react by registering all of their shares prior to placement. H & S postulated that portions of the discount were due to the higher required assessment and monitoring costs required of private placement investors and the tendency of the market to bid up the price of traded shares where private placement investors had taken an interest and shown a willingness to invest.

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<sup>33</sup> Hertz, Michael and Richard Smith, "Market Discounts and Shareholder Gains for Placing Equity Privately", *Journal of Finance*, 48 (1993), 459-485.

## D. Summary of DLOM Studies/Methods Analytical Approaches

H & S analyzed 106 private equity placements with about 75% of those being firms traded over-the-counter. The time period of their study was January 1, 1980 through May 31, 1987. The measurement date used was 10 days after the announcement of the placement was made. Of the placements analyzed, 45 involved registered shares, 18 involved unregistered shares and 43 had an unknown registration status. H & S assumed that all of the placements where the registration status could not be determined were, in fact, registered for study purposes since this would lead to a conservative result with regard to the discount differential.

A regression analysis was performed using 7 independent variables with the registered versus unregistered variable used to estimate DLOM. The average private placement discount overall was found to be 20.14% with about two-thirds of that discount (13.5%) being related to concerns about lack of liquidity. The remainder of the discount was due to such other factors as the size of the placement, the degree of financial distress existing in the firm and the nature of the placement buyers. In the context of their paper, H & S seem to be using the terms marketability and liquidity interchangeably rather than with the type of differentiation that was noted earlier per Pratt and Abbott. As stated above, H & S considered this to be an upper bound for DLOL/DLOM due to the perceived difference in assessment and monitoring costs between registered and unregistered shares.

### Strengths

The strength of the H & S study is that it is somewhat more complete in its analysis than the Wruck study as seven variables potentially affecting discounts are identified and analyzed using a regression model. Through the multi-variate analysis, H & S were able to isolate what they believe to be the specific effect of lack of marketability from the effects of the other variables considered. This effect is measured at 13.5% based on the use of a dummy variable relating to registration status.

- The H & S sample is primarily (75%) made up of smaller companies that are traded over-the-counter whereas the Wruck sample was composed of companies traded on major stock exchanges. These smaller companies would seem to be more like the companies that are most often the subject of valuation assignments where lack of marketability is a concern than are the larger companies studied by Wruck.

### Weaknesses

- The H & S study once again suffers from sample selection, registration status determination and measurement point problems as was the case with the Wruck study.

## D. Summary of DLOM Studies/Methods Analytical Approaches

- H & S were able to determine the registration status of only 63 of their 106 sample companies and assumed that the 43 that could not be determined would all be considered as registered. This is an obvious, very serious problem with the methodology employed since it is the registration status variable that is put forth as the measure of lack of marketability in the study. H & S consider this approach to lead to a conservative result since the assumption used would act to reduce the amount of discount attributable to lack of marketability.
- H & S chose a measurement point at 10 days after the announcement date which gives any bounce upon announcement some time to dissipate prior to the measurement. This choice should also act to produce a more conservative discount result; however, the choice of measurement point remains arbitrary and totally subjective.

### Important Parameters of the Study

The analysis parameters considered by H & S are:

- Fraction of total outstanding stock placed
- Financial distress of issuer<sup>34</sup>
- Book to market ratio of stock value
- Log of the proceeds of the offering<sup>35</sup>
- Registration status
- Investor is an individual
- Investor is a member of management

The other important variables in the approach are the sample selection methodology and the choice of measurement point.

### View of the Valuation Community

The H & S study, like the Wruck Study, has been cited by a number of practitioners but is basically utilized as background material to introduce the subject of investigating marketability discounts analytically. Although the average and median discounts of the study are offered as evidence that the results of the benchmark studies may be much too high, the H & S discounts themselves are generally not offered as actual discount proposals.

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<sup>34</sup> The financial distress parameter is based on an analysis of such things as the company's solvency, liquidity, return on assets, debt-serving capacity, etc. to measure overall financial condition.

<sup>35</sup> This is a measure of the size of the placement in dollar terms expressed on a logarithmic scale

## D. Summary of DLOM Studies/Methods Analytical Approaches

### View of the Courts

Since the numerical values of the H & S discounts have not been advanced in court as the primary determiners of proposed discount amounts, the courts have not specifically opined on the H & S study and its results.

### (c). Bajaj, Denis, Ferris and Sarin<sup>36</sup>

#### Background and Summary

Bajaj et al set about to study the concept of firm value and marketability discounts. They defined marketability as how quickly an asset can be converted into cash, without the owner incurring substantial transaction costs or having to give significant price concessions. They postulated that lack of marketability increases opportunity costs for asset holders and that such holders are also exposed to increased risks of loss. Both of these factors increase risk and lead to the need for discounting to lure investors to buy.

Bajaj et al set out the following factors affecting marketability:

- Uncertainty of the assets value
- Lack of availability of information on the asset to an outsider
- Availability of close substitutes for the asset
- Duration of the restriction on trades of the asset
- Size of the block being sold

An analysis was made of private equity placements during the period January 1, 1990 through December 31, 1995 involving 88 transactions. The measurement date used was 10 trading days after the announcement date. Accounting data was drawn from Compustat. A cross-sectional analysis of discounts was made using regression techniques.

Bajaj et al found that, on average, all private placements are made at discounts whether the block placed consists of registered shares or non-registered shares. For registered shares, the average discount was 14.04% and for unregistered shares the average discount was 28.13%. The respective median discounts were

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<sup>36</sup> Bajaj, Mukesh, David J. Denis, Stephen P. Ferris and Atulya Sarin, "Firm Value and Marketability Discounts", *Journal of Law and Economics* (2002).

## D. Summary of DLOM Studies/Methods Analytical Approaches

9.85% and 26.47%. Combining unregistered and registered share transactions gave an overall average discount of 22.21% and a median discount of 20.67%.

As a first estimate of DLOM the average discounts were compared to get a discount differential of 14.09%. This was predicated on the premise that no DLOM should exist for registered shares since such could become immediately freely traded. A regression analysis was then conducted to attempt to further sort out the factors contributing to the overall discounts. This analysis used four independent variables with the registered/unregistered status being one. The coefficient for the registration variable turned out to be 7.23% indicating that registered shares would require a lesser discount than unregistered shares by that amount. This provided a more refined estimate of the specific effects of lack of marketability in Bajaj's view.

Bajaj et al also stratified their overall discount data to provide statistics for the larger group of discounts, the middle group of discounts and the smallest group of discounts. Averages of 43.33%, 20.36% and 2.21% were derived by group. Discussion was provided of the various factors that might explain the range of differences among these stratified groups. These included the fractional size of the block to total shares outstanding, the business risk facing the firm, the degree of financial distress of the firm and the total proceeds raised in the offering.

The Bajaj et al study has generated considerable response and criticism as it was the first study offered as a basis for court testimony for tax purposes when Dr. Bajaj began testifying before the Tax Court in cases such as the *Estate of Gross* and *McCord*. Most notable among the parties criticizing the study were Shannon Pratt, Mark Mitchell, Lance Hall and Chris Mercer. These critics found problems with many facets of the study including sample selection, measurement date, the combining of registered and unregistered share transactions, the choice of regression variables, the failure to consider the holding period as an explanatory factor, the failure to consider the Rule 144 affiliate provisions, the failure to properly identify registration status, and lack of rigor in the regression model employed.

### Strengths

- The Bajaj study like the H & S study concentrated primarily (82%) on companies traded over-the-counter. Although there is some debate among critics, the Bajaj sample of 88 companies seems to be better defined and the registration status of the component private placements more confidently determined with about 58% being unregistered.

## D. Summary of DLOM Studies/Methods Analytical Approaches

- As mentioned for H & S the use of primarily smaller companies seems to be better suited to the measurement of the effects of lack of marketability than companies traded on major exchanges.
- Bajaj considered five different parameters that were seen to affect discounts, one of which was a variable based on registration status. This approach, like the approach pursued by H & S, allowed a direct measurement of what Bajaj considered to be the effects of lack of marketability. His isolated discount amount of 7.23% is supported by his model but seems to be too low to survive the application of a sanity check.

### Weaknesses

The potential weaknesses of the Bajaj study have been spotlighted by a number of its critics including Pratt, Hall, Mercer and Mitchell and Norwalk. These weaknesses are concentrated in the areas of concern over sample choice, the remaining presence of some uncertainty in actual registration status, the relatively low coefficient of determination or  $R^2$  factor<sup>37</sup> generated by the regression model used and the choice of a measurement date of 10 days after the announcement.

- Certain writers have pointed to data errors in the sample and the failure to consider other transactions occurring within the analysis period that are considered to be logical choices with required data available.
- There is some question among analysts as to what the 7.23% discount amount attributable to lack of marketability by Bajaj really measures and whether, even if it truly measures a pure marketability component of discount, it is the proper level of discount to be considered in a transactional analysis. Bajaj himself has been somewhat inconsistent in how he applies the results of the study using the 7.23% in certain cases and a larger discount that is said to include the effects of assessment and monitoring costs in other cases.
- Another weakness of the Bajaj study in the view of his critics is it does not explicitly consider the length of the required holding period for an unregistered placement as a factor in the analysis. Not all unregistered placements are subject to the same holding period limitations and, accordingly, the analysis of registered versus unregistered placements should not be treated as a binary variable as Bajaj has proposed.

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<sup>37</sup> The coefficient of determination is a measure of how well a regression model fits the data by indicating how much of the total data variation is explained by the model. If all the data were to fall directly on the model line then the coefficient would be 1.00. The lower the coefficient the less of the variability of the data is explained by the model.



## D. Summary of DLOM Studies/Methods Analytical Approaches

- Finally, critics argue that simply because some private placement shares are registered does not automatically make them freely tradable such that no DLOM should apply to them.

### Important Parameters of the Study

Bajaj combines the five areas potentially affecting marketability related discounts into four parameters for use in his model. These parameters are:

- the percent of shares placed out of the total outstanding shares,
- the Z-score<sup>38</sup> which is a measure of a firm's financial strength or lack thereof based on an analysis of ratios focusing on solvency, liquidity, return on assets, debt serving capacity, etc.,
- the registration status of the placement and
- the volatility of the stock as determined using actual data for the publicly traded stock of the sample company.

Other important variables are the selection of the sample to be analyzed and the choice of the measurement point.

### View of the Valuation Community

Unlike Wruck and Hertzel & Smith, the Bajaj study has received intense attention from the valuation community, much of it critical in nature. Critics such as Pratt, Hall, Mercer and a number of other practitioners have cast much skepticism on Bajaj's sample selection, his model's weaknesses including its rather low explanatory value as measured by  $R^2$ , the use of registration status as a binary variable rather than one that considers the differential effects of required restriction periods and the unreasonably low amount that is attributable to lack of marketability as a discrete variable. The general thesis advanced by his critics for Bajaj's relative success in his court appearances is that he had poor and unprepared opposition that could not and did not exploit all of the weaknesses in his study and his testimony.

The critics advance a number of reasons why the Bajaj approach should not be accepted by practitioners but, in each case, the criticism is accompanied by support for the critics own preferred approach to DLOM. In the case of Pratt and Hall, this is the use of the benchmark study approach while in the case of Mercer it is the use of his QMDM approach. Hall believes that the data in the FMV

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<sup>38</sup> See Edward I. Altman, *Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy*, J. Fin 589, 589-605 (1968). The higher the Z-Score of a company, the stronger its financial position.

## D. Summary of DLOM Studies/Methods Analytical Approaches

Opinions restricted stock study can be used to counteract the conclusions that Bajaj has advanced.

### View of the Courts

To date, only Bajaj and his colleague Dr. Shapiro have gone to Court with the analytical approach as their main support for a DLOM discount selection. Bajaj has testified in the *Estate of Gross*<sup>39</sup>, *Litman and Diener v. USA*<sup>40</sup>, *McCord et ux v. Commissioner*<sup>41</sup> and *Richie C. Heck v. Commissioner*<sup>42</sup> among others. Shapiro utilized the same approach in his testimony in *Lappo v. Commissioner*<sup>43</sup>.

In general, the Courts have given favorable treatment to Bajaj's general approach to DLOM citing the conceptual basis and the use of mathematical techniques to separate out contributing factors. However, no Court has accepted his 7.23% estimate as the proper DLOM at face value. In *McCord*, the Court instead chose to look at all of the Bajaj data and to select a DLOM based on the summary results from his middle strata of discount transactions arriving at a number of 20%. A similar approach has been taken in other cases where the 20% discount has been accepted as a starting point and then adjusted up to 23% or 25% based on factors that the Court thought were important. In *Gross*, Bajaj did not propose a strict DLOM discount based on his study but instead argued for 25% which included a 20% original amount plus 5% to account for the S corp. effects on marketability. This total discount was accepted by the Court.

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<sup>39</sup> *Estate of Gross*, T.C. Memo 1999-254, 78 T.C.M. (CCH) 201, T.C.M. (RIA) 99254, 1999 Tax Ct. Memo LEXIS 290

<sup>40</sup> *David S. and Malia A. Litman v. The United States*, United States Court of Federal Claims, 2007 U.S. Claims LEXIS 273, August 22, 2007

<sup>41</sup> *McCord v. Comr.*, 120 T.C. 358 (2003)

<sup>42</sup> *Heck v. Comr.*, T.C. Memo 2002-34

<sup>43</sup> *Clarisa W. Lappo v. Comr.*, T.C. Memo 2003-258, Tax Ct. Memo LEXIS 257, 86 T.C.M. (CCH) 333

## D. Summary of DLOM Studies/Methods Analytical Approaches

### (d). Ashok B. Abbott<sup>44</sup>

#### Background and Summary

Abbott studied empirical methods for estimating marketability and liquidity discounts. He defines marketability as the ability to sell a block of securities in an established and efficient public capital market, with relatively low transaction costs, and with minimal effect on that security's public market price. Liquidity is then seen as the ability to convert a block of securities into cash. Per Abbott, marketability refers to a right and liquidity is a measure of speed.

Abbott believes that neither the pre-IPO nor the Restricted Stock Studies give very usable results for a number of reasons. Among these are changes in the Rule 144 holding period, the growth of the derivatives market, the reduction in required trading costs due to discount brokerages, the new transparency rules established by the SEC and FASB, and their use of averages.

Per Abbott, recent law changes and market developments have made public markets more liquid but this change does not extend to private markets which could lead to an understatement of discounts appropriate to these markets. Hence, he believes that a more scientific and statistically supportable approach to marketability and liquidity is now required. He then analyzes a number of studies and discount indications that exist. This review provides the following lack of liquidity indicators.

- In 1996, for shares traded on the NYSE, the most liquid stocks when compared to the least liquid stocks indicated a discount for lack of liquidity range of 35.5%
- In an IPO study for the period 1993 to 2003, the average trimmed mean<sup>45</sup> for DLOL is 6.05% based on some 7,824 IPOs
- In a 2004 IPO study the range for DLOL was from 4.3% to 9.9% depending upon market capitalization

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<sup>44</sup> Ashok B. Abbott, Various Dates, *Empirical Measures of Marketability and Liquidity Discounts, Discounts for Lack of Marketability: An Empirical Analysis* and *DLOM – Concepts and Models*, Presentations at Various ASA and NACVA Conferences and on the BVR Teleconference of April 26, 2006

<sup>45</sup> The trimmed mean is the average obtained from a subset of the data after some of the largest and the smallest values have been removed. It is thought to be a better measure of the central tendency of the concentrated core of the data than is the overall mean.

#### D. Summary of DLOM Studies/Methods Analytical Approaches

- Small cap stocks had greater holding periods during the period from 1993 to 2004 in the range of 30 to 130 months as compared to 10 to 21 months for large cap stocks; the overall average range was from 25 to 69 months
- Large cap stocks have been as much as 9 times more liquid than small cap stocks for trades during 2001

Abbott mentions that Longstaff<sup>46</sup> has postulated a DLOL of 3% to 42% based on his option study where 3% is for a 5% block size for a large cap stock and 42% is for a 25% block size for a small cap stock. These discounts are upper-bounds since Longstaff assumes perfect market timing in making his analyses.

Finally, Abbott concludes that for smaller block sizes a proper DLOL is limited to less than 25% with the DLOL for a 5% block limited to about 15%.

#### Strengths

Abbott recognizes the differences between public and private markets and the importance of block size as considerations in the discount for lack of marketability. He further recognizes the effect of relatively new innovations in security markets as factors that act to reduce the discount required by investors in non-liquid stocks. Further, he recognizes the importance of overall company capitalization on the holding periods required to sell stocks. Abbott cautions as to the risks involved in using public stock-based discounts for stocks that are not publicly traded on any recognized exchanges. His strengths are mostly conceptual rather than of a nature that would necessarily lead to a reliable numerical estimate for either DLOL or DLOM.

#### Weaknesses

Abbott's results have been mostly presented in academic and valuation society environments and have not been properly vetted by either practitioners or the courts. It is doubtful that his work could serve as a primary approach to marketability quantification as of the present time.

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<sup>46</sup> Francis A. Longstaff, *How Much Can Marketability Affect Security Value?*, J. Fin., 1767, December 1995

## D. Summary of DLOM Studies/Methods Analytical Approaches

### Important Parameters of the Study

Variables indicated by Abbott as potentially significant factors in liquidity and marketability include

- block size,
- overall market capitalization,
- availability of hedging opportunities,
- anticipated holding period of market participants and
- the general need for liquidity in the economy in general.

### View of Valuation Community

To date, the valuation community has shown an interest in Abbott's work and his concepts. He has been invited to speak at numerous valuation conferences and to participate in a number of panel discussions such as the BVR Teleconferences. There has been no use of the results that he has generated as a basis for discounts that would properly serve as a foundation for an overall valuation.

### View of the Courts

The Abbott analyses and findings have had no vetting in the courts.

### Conclusion

Overall, many judges seem to be using the work of Bajaj and the other analytical studies as ammunition to hold all practitioners accountable for unsupported reliance on the benchmark studies. Even though an acceptable bottom line number has not come out of these studies per se, they have raised several questions and have tended to show that the benchmark studies can sometimes lead to unreasonably high results. Among the questions that they have brought to the surface is the existence of investors with long investing horizons for which marketability is not a particular concern and the development of the derivative markets which have allowed the creation of synthetic liquidity that did not exist at the times when the benchmark studies were constructed.

As a result of the weaknesses cited relating to sample selection, sample point classification and measurement point concerns, it is unlikely that these approaches can be used to derive a numerical result that will go forth unchallenged. Instead, the raw data collected and the many component factors proposed can be used to make subjective judgments about discount magnitudes that would seem more satisfactory than using the gross averages generated by

#### D. Summary of DLOM Studies/Methods Analytical Approaches

the benchmark studies, either with or without unsupportable adjustments for changing facts and circumstances. For example, consideration of volatility and expected holding period as opposed to restriction period would seem to be factors that provide meaningful insight to the DLOM and DLOL question. Also the availability of hedging strategies can act to increase effective liquidity where those strategies exist. These strategies replicate the existing value parameters of a non-liquid security by combining the value parameters of other securities that are publicly traded and, therefore, more liquid.

## 4. Other Approaches

### (a) QMDM (Christopher Mercer)

#### Background and Summary

The complete reference for this approach to the Discount for Lack of Marketability (“DLOM”) is the “Quantitative Marketability Discount Model” (see Mercer’s book: Quantifying Marketability Discounts, by Z. Christopher Mercer, ASA, CFA, *Peabody Publishing, LP*, 1997). The model calculates a matrix of discounts for lack of marketability, based upon a range of variables. Variables include rate of appreciation in assets, holding period until liquidation, and required rate of return to the hypothetical investor. The appraiser estimates which variables from the matrix are most appropriate for the subject interest. The intersection of those variables within the calculation matrix yields the DLOM.

Given the variable inputs, discounts from this method can vary significantly. For example, a “base case” illustration on page 225 of Mercer’s book presents a matrix of possible discounts ranging from 5% to 100%. Within the matrix, three discounts are proposed for purposes of discussion (31% [low], 58% [medium], and 71% [high]).

#### Areas of Focus

In discussing this approach with taxpayer or taxpayer’s appraiser, the following areas of focus should be explored:

- On what basis has the Taxpayer’s appraiser estimated the expected rate of appreciation (i.e. growth) on the underlying investment assets?
- On what basis has the Taxpayer’s appraiser estimated the holding period before the hypothetical buyer would receive the cash flow return on their investment?
- On what basis has the Taxpayer’s appraiser estimated the required rate of return to the hypothetical buyer?
- How does the required rate of return compare to alternative investments that were available to the hypothetical buyer on the valuation date?

## D. Summary of DLOM Studies/Methods Other Approaches

### Strengths

- As its name states, the QMDM provides a quantitative basis for reaching an opinion of the DLOM.
- Instead of arbitrarily selecting “35%” after a vague discussion of valuation theory and restricted stock studies, the QMDM allows the appraiser to estimate specific factors (e.g. rate of appreciation, holding period, and required rate of return), to conclude a specific DLOM from the calculation matrix.

### Weaknesses

- While it avoids arbitrary selection of a DLOM, estimation of factors for the calculation matrix can be just as arbitrary and subjective.
- The matrix increases the number of things the appraiser needs to have an “opinion” about, potentially leaving the appraiser over-extended on their clairvoyance about a multitude of events expected to occur many years into the future.
- Alternatively, if the appraiser simply relies upon management (i.e. the “client”) projections for parameter estimation, the appraiser’s opinion of the DLOM might lack credibility and independence.

### Important Parameters for this Approach

- 1) Base value of the marketable minority interest (the base value would be the pro rata of the subject interest, after the discount for lack of control, but before the discount for lack of marketability);
- 2) Expected appreciation on base value over the holding period;
- 3) Expected dividend yield over the holding period;
- 4) Expected growth rate in dividends over the holding period;
- 5) Assumed length of the holding period in years; and
- 6) Required rate of return to hypothetical investor over the holding period

Variations of the QMDM can incorporate additional factors, such as interim cash flows, compensation to officers (including over-compensation), taxes, etc. However, the model can become very complicated with the introduction of additional variables.



## D. Summary of DLOM Studies/Methods Other Approaches

### Prevalence in professional practice:

This approach has seen minimal use by outside valuation professionals as the primary basis for the DLOM. More recently, QMDM has been presented as additional support (“sanity check”) for a DLOM estimated using methods other than the QMDM.

From the perspective of originator of QMDM method: At the September 18, 2008 DLOM Summit in San Diego, Chris Mercer, the originator of QMDM, argued in favor of this approach, stating that his firm still uses the QMDM (although he mentioned that E&G tax valuations were not a significant portion of his firm’s case work). He further stated that, despite some Court rulings involving the QMDM approach, he (and his firm) has not been rebutted in Court for using the QMDM.

### View of the Courts

Prior to the QMDM, the Courts had criticized appraisers for a lack of quantitative basis for their DLOM determination (e.g. arbitrarily selecting a 35% DLOM). The QMDM appeared to be an answer, but *Weinberg v. CIR* (T.C. Memo. 2000-51) and *Janda v. CIR* (T.C. Memo. 2001-24) suggest otherwise. In each of these cases, the QMDM was criticized:

*Weinberg v. CIR*: “We disagree with the discount computed by Dr. Kursh on the basis of the QMDM model because slight variations in the assumptions used in the model produce dramatic differences in the results.”

*Janda v. CIR*: “We have grave doubts about the reliability of the QMDM model to produce reasonable discounts...”

## (b) NICE (William Frazier)

### Background and Summary

According to Howard, Frazier, Barker, Elliot, Inc. ([www.hfbe.com](http://www.hfbe.com)), a Texas-based valuation firm, firm principal William Frazier authored an article, "Non-marketable Investment Company Evaluation (NICE)", that appeared in the November/December 2006 issue of *Valuation Strategies* (Vol 10, No 2, published by Warren, Gorman & Lamont, RIA Group, Boston MA).

## D. Summary of DLOM Studies/Methods Other Approaches

NICE is a valuation system under the income approach designed to determine the fair market value of equity interests in closely held investment entities. NICE uses investment returns to calculate value.

According to the article in Valuation Strategies (see above for citation reference):

[NICE] is a valuation system under the income approach to value. It is designed specifically to determine the fair market value of equity interests in closely held investment entities, such as family limited partnerships, S corporations, and limited liability companies. NICE does not use [lack of control and lack of marketability] discounts in its operation. Instead, lack of control and lack of marketability are viewed as investment risks embodied in the required rate of return for the subject interest.

The key issue is that incremental rates of return for lack of control and/or lack of marketability need to be estimated. Such estimates can begin to appear subjective, depending on the availability of adequate information.

Given the variable inputs, discounts from this method can vary. For example, a “scenario” illustration on page 46 of the respective issue of Valuation Strategies shows a discount range from 37% to 47.5%, with an indicated discount of 42.25%.

The NICE method specifically states that it should not be used when the holding period is either known or can be reasonably estimated. According to the article, the method assumes a “very long-term and illiquid investment”... “[T]he liquidation date can be a very distant event, with a practical range of no less than ten years.”

Thus, the NICE method would tend to lead to an elevated estimate of total discounts for an interest that did not meet these assumed conditions.

### Areas of Focus

In discussing this approach with the taxpayer or taxpayer’s appraiser the following areas of focus should be explored:

- On what basis has the Taxpayer’s appraiser estimated the additional rate of return to compensate for lack of control and/or lack of marketability?
- Do the incremental rates of return reflect arbitrary selection of 1%, 2%, 3%, etc.?

## D. Summary of DLOM Studies/Methods Other Approaches

- How does the required rate of return compare to alternative investments that were available to the hypothetical buyer on the valuation date with comparable degrees of total risk?

### Strengths

- The NICE method avoids subjective estimation of discounts for lack of control and lack of marketability. On that basis, the method appears to be a more traditional, and straight-forward, “income approach” to valuation.
- NICE is also presented as a better alternative to the QMDM, on the basis that the QMDM does not specifically address the additional required rate of return for lack of control and/or lack of marketability, while the NICE method does.

### Weaknesses

- The method assumes the hypothetical buyer can demand (and receive) a higher required rate of return for lack of control and lack of marketability.
- The basis (or capability) for the hypothetical buyer to receive the higher rate of return can become a matter of subjective estimation.
- For example, the method claims to avoid subjectivity, but the illustration within the article estimates (somewhat subjectively) a precise 2.00% increase in required rate of return for lack of marketability. This incremental return is illustrated on the basis of above-average performance of certain mutual funds. However, the article admits that mutual funds generally cannot maintain above-average performance indefinitely.
- The method assumes a holding period well-in-excess of 10 years (upwards of 50 years in some examples). However, it could be argued that “predicting” a liquidation date 50-years into the future is just as speculative as subjectively estimating a DLOM.

## D. Summary of DLOM Studies/Methods Other Approaches

### Important Parameters for this Approach

- 1) Baseline rates of return for market interest rates and stock market returns (baseline reflects comparable investments that do not have additional risks from lack of marketability); and
- 2) Incremental returns for lack of control and lack of marketability, which are estimated on the basis of various “spreads” of different types of investments.

### Prevalence in Professional Practice:

The NICE or Frazier method has not been seen in valuation reports we have reviewed. However, valuation reports have used rate-of-return methods to value closely held investment interests (or as “sanity checks”), but the terms NICE or Frazier were not cited.

William Frazier continues to conduct seminars on his “NICE” method. For example, he presented on NICE at the AICPA/ASA conference in Las Vegas in November 2008.

### What the Courts say about this model

No Court references were found for the NICE or Frazier method. If presented to a Court it is likely that the Court will criticize this method as relying upon subjectively-estimated incremental rates of return for lack of control and lack of marketability unless some definitive market evidence were provided in support of these rates.

## (c) NERA (David Tabak)

### Background and Summary

NERA (National Economic Research Associates) is a consulting firm. Dr. David Tabak, a Senior Vice President with NERA, published an in-house working paper entitled: “A CAPM-Based Approach to Calculating Illiquidity Discounts”. The working paper is dated November 11, 2002, and is posted for free on NERA’s website ([www.nera.com](http://www.nera.com)).

## D. Summary of DLOM Studies/Methods Other Approaches

According to the NERA website ([www.nera.com](http://www.nera.com)):

In this working paper, Dr. Tabak provides a review and analysis of existing studies and theories on calculating appropriate illiquidity discounts for restricted stock. Dr. Tabak discusses how existing studies have limited applicability in calculating an appropriate discount because they generally present only median or average results.

As an alternative, Dr. Tabak offers a theoretical model based on the CAPM, or capital asset pricing model, that allows for a quantification of the illiquidity discount based on objective criteria specific to the asset under consideration. This equity risk premium-based model is the first approach to apply the CAPM to the process of calculating illiquidity discounts, and offers a number of benefits over using simple average discounts or any of the other methodologies discussed in this paper.

The result is a framework for measuring illiquidity discounts that vary over time and depend on the length of the restriction and the riskiness of the illiquid asset. Perhaps most importantly, Dr. Tabak's new model is less subjective than the analysis often used in practice today.

Given the variable inputs, discounts from this method can vary. For example, page 16 of the working paper presents a table of "Implied Illiquidity Discounts" (based on different equity risk premiums). The table indicates a full dataset range of discounts from 15.4% to 82.9%, with mid-point average discounts ranging from 37.8% to 44.8%.

### Areas of Focus

In discussing this approach with taxpayer or taxpayer's appraiser, the following areas of focus should be explored:

- For reference: The higher the equity risk premium (or the greater the expected volatility of returns relative to the overall market [i.e. "Beta"]), the lower the estimated value (and vice-versa). (Briefly defined: Beta is a factor indicating the relative risk of a specific investment, as compared to overall risk attributable to the aggregate market of investments.)
- How was the additional equity risk premium (or Beta) for lack of control and/or lack of marketability determined?

## D. Summary of DLOM Studies/Methods Other Approaches

- How was the time period until liquidation of the initial investment determined?
- How does the equity risk premium (or Beta) compare to alternative investments that were available to the hypothetical buyer on the valuation date? (NOTE: While some methods compare overall rates of return [e.g. risk-free rate plus the equity risk premium], the NERA method is focused upon the equity risk premium portion of the return.)

### Strengths

According to the article,

- the method provides a quantitative basis (using the capital asset pricing model) to incorporate lack of marketability as an additional “risk” that increases the equity risk premium (and lowers the “price”);
- “The calculation of an illiquidity discount is objective (or at least relatively so) because it can be calculated based on volatility (actual for a security, from a peer group for a company) and the equity risk premium.”

### Weaknesses

- The method requires that a number of variables be either measured from market activity, or estimated from market comparables. Estimates based upon selected market comparables can introduce subjectivity into the valuation analysis.
- Additional estimates with respect to holding period can introduce further subjectivity into the valuation analysis.
- The model is theoretical in nature and there is no sound way to calibrate its results against the market. Tabak has run a number of analyses against S & P 500 stocks for various years and has used the old benchmark study averages to provide a sanity check on his results.

### Important Parameters for this Approach

According to the working paper, the model requires data (or estimates) of:

- 1) risk-free rate of return;
- 2) expected return to a market portfolio;
- 3) expected return to the subject asset;

#### D. Summary of DLOM Studies/Methods Other Approaches

- 4) covariance of the subject asset with respect to the market portfolio;
- 5) standard deviation of rates of returns; and
- 6) period of time that the asset is restricted from sale.

Of particular significance to the use of this method, the working paper (see above for citation reference) states:

To begin, assume that these quantities are all measurable...[T]his theory will still require a somewhat subjective analysis if one or more of these quantities, typically T, the time of the restriction, must be estimated based on qualitative data.

This suggests (from Dr. Tabak himself) that there are inherent weaknesses in the method.

#### Prevalence in Professional Practice:

We have not seen the NERA or Tabak methods, per se. However, some appraisals have used CAPM-based methods to estimate a risk-adjusted rate of return for non-marketable securities (or as “sanity checks”).

#### What the Courts say about this model:

No Court references were found for the NERA or Tabak method. It is deemed likely that the Courts might criticize this method as being overly-complicated, and/or relying upon subjectively-estimated variables.

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### (d) Partnership Profiles (Partnership Spectrum)

#### Background and Summary

Partnership Profiles (aka “Partnership Spectrum” or “Direct Investments Spectrum”) is a quarterly publication (moving to an online database [www.partnershipprofiles.com or www.dispectrum.com]) that summarizes data on re-sales of minority interests in Real Estate Limited Partnerships (“RELP’s”). Partnership Profiles reportedly tracks more than 300 different RELP’s. Data for each re-sale includes a pro rata net asset value attributable to each RELP interest being sold. On that basis, a “discount” from pro rata net asset value can be inferred from each re-sale.

Partnership Profiles is primarily used as the basis for lack of control discounts on minority limited partnership interests. However, the RELP re-sale market is so small (i.e. “thinly-traded”) that Partnership Profiles data arguably reflect some additional consideration for lack of marketability. With respect to lack of marketability, Direct Investments Spectrum has stated:

Although it is not possible to precisely quantify the amount of discount attributable to marketability versus lack of control considerations, it is the opinion of Direct Investments Spectrum, along with many appraisers, that most of the overall discount is due to lack of control issues.<sup>47</sup>

NOTE: Real Estate Investment Trusts (aka “REIT’s”) are similar to RELP’s, and are also commonly-cited as the basis for lack of control discounts on minority interests. However, REIT’s are freely-traded in an active market, and therefore, discounts observed from REIT’s are generally assumed to exclude any consideration for lack of marketability.

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<sup>47</sup> May/June 2004 issue of *Direct Investments Spectrum*, at www.dispectrum.com.



## D. Summary of DLOM Studies/Methods Other Approaches

### Areas of Focus

In discussing this approach with taxpayer's or taxpayer's appraiser, the following areas of focus should be explored:

- Are the Partnership Profiles comparables similar to the subject interest in terms of: a) the type of real estate; b) relative debt ratios; and c) cash distributions yield?
- Has Taxpayer's appraiser been able to verify if the baseline net asset values in Partnership Profiles data were established using actual "appraisals", versus management estimates of the values of underlying real estate investments?
- Assuming that Taxpayer's appraiser used Partnership Profiles to estimate the discount for lack of control, and then used another method (e.g. restricted stock studies) to estimate the DLOM, did Taxpayer's appraiser give any consideration to "lack of marketability" considerations that already exist within Partnership Profiles data (i.e. to avoid double-counting lack of marketability factors)?
- In the case of a Charitable Contribution appraisal, does it appear that Taxpayer's appraiser is trying to minimize the discount (and maximize value) by using Partnership Profiles for a single, combined discount for lack of control and lack of marketability?

### Strengths

- Appraisers who wish to avoid "over-discounting" might rely on Partnership Profiles data to provide a single "combined discount" for lack of control and lack of marketability.
- Partnership Profiles data include descriptive factors of the types of RELP investments, debt ratios, and whether or not the RELP has been making regular cash distributions to limited partners. These factors provide specific bases to identify comparables within the data.

### Weaknesses

- It has been argued that RELP's referenced in Partnership Profiles are not representative or not comparable to subject interests being valued in appraisal reports (i.e. RELP's are not directly comparable to family limited partnerships).

## D. Summary of DLOM Studies/Methods Other Approaches

- The data have also been criticized as being inconsistent with the fair market value standard. One reason is that Partnership Profiles data reportedly reflect remnant RELP's formed in the 1970's under different tax laws. Those RELP's no longer provide the same tax benefits after tax law changes in 1986. On that basis, Partnership Profiles might reflect out-of-favor investments, being sold under distressed conditions at high discounts to net asset value.
- The pool of RELP's is also reportedly shrinking, creating a potential problem of statistical significance in the quantity of reported sales for each type of RELP.
- Another criticism is that the pro rata net asset values attributed to RELP re-sales might have been arbitrarily reported (i.e. management estimates), and do not reflect thorough appraisals of underlying investment assets within the RELP's.
- Method is logically limited to entities that have substantial amounts of real property assets in their portfolios.

### Important Parameters for this Approach

- 1) Types of underlying investments (e.g. real estate, vacant land, etc.);
- 2) Relative debt ratio; and
- 3) Dividend yield on net asset value.

### Prevalence in Professional Practice

Partnership Profiles data are primarily used to estimate lack of control discounts on minority limited partnership interests. In most cases, the appraiser would use Partnership Profiles to estimate the lack of control discount, and then use another method (such as restricted stock studies) to estimate a separate DLOM. However, because of the nature of the data, some appraisers use Partnership Profiles to estimate a single combined discount for lack of control and lack of marketability.

In cases of charitable contributions (e.g. charitable remainder trusts), the appraiser might cite Partnership Profiles as the source for a single, combined discount for lack of control and lack of marketability (e.g. if an appraiser were seeking to avoid over-discounting).

## D. Summary of DLOM Studies/Methods Other Approaches

What the Courts say about this Approach:

In *Estate of W.W. Jones II v. CIR* (116 T.C. No. 11 filed March 6, 2001), the taxpayer's expert acknowledged that: "[A] large discount for lack of marketability is already built into the secondary market discount [from Partnership Profiles data]." The Court agreed and reduced the taxpayer's [separately-determined] lack of marketability discount from 20% to 8%.

In *Estate of Kelly v. CIR* (T.C. Memo 2005-235), the Court stated: "We are also not persuaded by ATI's analyses of PPI's studies regarding minority discounts as ATI admits that these discounts contain some element of discount for lack of marketability, and therefore these studies result in an overstatement of the minority discount."

In *Lappo v. CIR* (T.C. Memo 2003-258), the Court stated: "[M]r. Oliver's reliance on the published RELP market prices seems questionable." The record in the Lappo case further noted that RELP's had very low trading volume, and that the underlying net asset values: "[R]epresent either estimates by general partners, appraised values determined by independent appraisers retained on behalf of the partnerships, or some combination of the two."

### (e) Public vs. Private P/E Ratios in Acquisitions (MergerStat)

#### Background and Summary

A dataset published in annual editions of *Mergerstat Review* sorts transactions into categories of "public" versus "private" companies being acquired [see *Mergerstat Review*, *FactSet Mergerstat, LLC*, 2007 (and earlier) edition(s), Table 1-12, page 20].

The table compares the "Median P/E" offered for public versus private companies, over a number of years. In general, Median P/Es offered in each year were higher for public companies (those whose shares were publicly traded at time of offer) than for private companies (those whose shares were NOT publicly traded at time of offer).

Based upon a premise that all data involved similar control conditions (all of the companies were being acquired), it would be reasonable to infer that the observed "premium" paid for public versus private companies reflected public

## D. Summary of DLOM Studies/Methods Other Approaches

company sellers' ability to liquidate their shares elsewhere—since a public market existed for those shares.

On that basis, public company sellers negotiated higher relative purchase prices because their shares were marketable, and could be sold elsewhere if necessary. However, private company sellers could not easily sell their shares elsewhere. Thus, they negotiated lower relative purchase prices because their shares lacked marketability. Observation of lower relative purchase prices for shares of private companies versus public ones implies a discount for lack of marketability ("DLOM").

While there is some variation of medians from year to year, the data indicate fairly-consistent median DLOM's by comparing public versus private acquisition Median P/Es in the range of 15% to 20%. The median has not followed any single trend in prior years. A review of the *MergerStat* data will indicate how the median has trended in the years leading up to any specific valuation date.

As a last consideration, these data reflect "control" conditions (all of the companies were being acquired). It could be argued that these data might be inappropriate for evaluating lack of marketability on "non-controlling" interests. Therefore, reliance upon these data for analyzing lack of marketability on a minority interest implies a condition that factors of "control" and "marketability" are effectively separable.

### Areas of Focus

In discussing this approach with taxpayer's or taxpayer's appraisers, the following area of focus should be explored:

- Assuming that Taxpayer's appraiser has referenced this method in relation to other methods (such as restricted stock studies), on what basis did the Taxpayer's appraiser weigh the significance of a DLOM from this method in relation to a DLOM from another method?

### Strengths

- Source data are objective, market-based transactions, and provide a simple illustration of the "discount" that sellers of closely-held companies had to accept (in arm's-length, market transactions) because they lacked an alternative to sell their shares in a public market.

## D. Summary of DLOM Studies/Methods Other Approaches

### Weaknesses

- DLOM's inferred from these data reflect median values for each year, and exclude any notation of upper- or lower-bound figures in each year's dataset. Summary median figures can obscure significant variance in underlying data.
- Variance in underlying data could reflect consideration of factors other than exclusively marketability versus lack of marketability.
- While it is possible to locate and analyze individual transactions in each year's dataset, a thorough analysis of variance (or comparability factors) in each year's dataset could be time consuming (some years included hundreds of transactions).
- Method is dependent upon specific companies acquired by public companies in a given year, and might not be reflective of DLOM levels in private transactions.
- The transactions summarized by *MergerStat* are control transactions, rather than minority interest transactions. This presents the question: "Is it reasonable to assume that the same P/E percentage comparison would apply to smaller traded interests?"

### Important parameters for this model

- 1) Since the data are summarized in the single table from *MergerStat Review*, there is no need to have anything other than a copy of the *MergerStat Review* covering the year of the valuation date.
- 2) The simplicity of this method prevents a more-thorough analysis of comparability factors. However, this method can provide additional support for an overall analysis of the DLOM that incorporates one or more additional methods of estimating the DLOM in a given appraisal.

### Prevalence in professional practice

The use of *Mergerstat* for DLOM has been seen a few times, but generally only as additional support for an overall DLOM analysis that used additional methods of estimating the DLOM. One possible explanation for its infrequency of use is that *MergerStat Review* is so commonly cited as the source of data for lack of

#### D. Summary of DLOM Studies/Methods Other Approaches

control discounts. Another citation of *MergerStat Review* in the DLOM area of the valuation analysis might be confusing to the reader.

What the Courts say about this model

No Court references were found regarding the use of *MergerStat* specific to estimating the discount for lack of marketability (there are Court rulings where *MergerStat* was used for the discount for lack of control). It is likely that the Courts will criticize this approach as being over-simplified and/or lacking comparability factors to support an opinion of the DLOM for a given subject interest (in cases where these data were used as the sole method for estimating the DLOM).

## ***E. Evaluation and Recommendations***

### **1. Approaching Marketability Discount as a Reviewer**

In considering the discount for lack of marketability as a reviewer, you will be presented with an approach and be concerned with judging its reasonableness, its reliability, its adherence to the prevailing facts and circumstances of the valuation problem at hand, its general acceptance within the valuation community and the treatment that the approach has received at the hands of the Courts. Hopefully, the taxpayer and/or the taxpayer's appraiser will have offered arguments for the approach or approaches chosen and for the numerical result decided upon. These arguments will need to be considered in detail and judged upon their merits. If the taxpayer or the appraiser has not offered any real analysis but rather simply presented a numerical result without substantial back-up that does not automatically make the result achieved wrong or unsustainable. You will need to analyze the result in the light of the prevailing facts and circumstances to determine whether it is reasonable or unreasonable.

If the result is considered unreasonable as a result of your review, you will likely be called upon by the client to produce an alternative independent estimate of DLOM based on your own analysis of the valuation problem. Your estimate should be constructed so as to not exhibit the same weaknesses found in the appraisal being reviewed. If the taxpayer or appraiser has used a valid approach or approaches but reached an unreasonable result you may be able to simply discuss what makes that result unreasonable and why you believe that your analysis yields a more reasonable result. If the taxpayer or appraiser has not used a valid approach in your view then you will have to start from scratch in preparing your opinion.

### **2. Approach Marketability Discount as a Valuator**

If you are approaching the question of DLOM fresh, either as a reviewer confronted with an unreasonable taxpayer position based on invalid approaches or as a valuator charged with making your own valuation discount decisions, it is often helpful to start with a basic question as relates to DLOM. That question is: "Under the prevailing facts and circumstances and considering the nature of the interest to be valued why is the DLOM not zero?" By enumerating the factors that would lead to a conclusion that some DLOM at all is appropriate you will be building a framework as to how substantial a discount for lack of marketability

## E. Evaluation and Recommendations

might be reasonable. This process will give you a reality check on DLOM amounts that you might ultimately derive using some of the approaches discussed in this job aid.

For example, if you have a very small minority interest in a non-publicly traded entity which has little or no history of interest sales and where shareholders are bound by a very restrictive shareholder's agreement, you could reasonably believe at the outset that a DLOM is appropriate and that it could be substantial. On the other hand, if you have a somewhat larger interest in a non-publicly traded entity that has a relatively large and active shareholder base with no restrictive shareholder agreement and where the potential seller holds a put right back to the corporation at fair market value then very little DLOM might be reasonable.

A common mistake among valuers considering DLOM (and discounts for minority interests) is to concentrate almost exclusively on the viewpoint of the hypothetical buyer who will be pushing at all times for larger discounts while ignoring the viewpoint of the hypothetical seller. In proposing a DLOM amount the valuator needs to ask whether this is an amount that a hypothetical seller could accept under the prevailing facts and circumstances and whether there is a reasonable chance that an arms-length negotiation between buyer and seller could arrive at such a discount amount. A fair market value determination requires the consummation of a hypothetical sale. If the analysis relies too heavily on the needs of the buyer it is likely that no such sale would occur and that this underlying premise of fair market value would be violated.

### **3. Dealing with Marketability Discount in a Report Review Under Certain Specific Situations – Typical Report Language for Getting Started**

Report reviewers frequently see the use of DLOM studies inappropriately. What follows is sample report language to use when these situations are encountered:

- a) Use of Pre-IPO studies to support DLOM
- b) Use of simple average or median from Restricted Stock Studies
- c) Use of analytical study results without getting behind the data
- d) Use of study results not supported by market data
- e) Reliance solely on court decisions



## E. Evaluation and Recommendations

### a) Use of Pre-IPO studies to support DLOM

The pre-IPO studies cited (Emory, Willamette or Valuation Advisors) examine the difference between pre-IPO stock transactions and their IPO price. When companies register for an IPO, they are required to disclose all transactions within three years prior to the offering. The pre-IPO studies observe transactions in privately-held companies that eventually completed an IPO. The private transaction price was compared to the public offering price, and the percentage discount from the public offering price is considered a proxy for the discount for lack of marketability.

These studies overstate DLOM and are unreliable for assessing the size of a discount for lack of marketability for many reasons:

- Because study data includes only successful IPO's, it artificially inflates the discount by ignoring unsuccessful IPO's
- The discount reflects more than lack of marketability—it includes risk that an IPO may not occur
- Almost always involve related-party transactions with employees or service providers who are compensated by a bargain price
- Pre-IPO transactions tend to be under priced as IPO's frequently involve high growth companies which are rapidly evolving (difference in pre & post company)
- Not contemporaneous – too much time gap exists between pre-IPO transaction and public offering
- There are indications that the *Willamette Management Associates Studies* 1999 and 2000 data may be skewed due to the dot.com "bubble"

#### Add if appropriate:

A business with the reputation, size and long history of profitability, such as \_\_\_\_\_ would likely have the option of becoming publicly traded with total floatation and registration costs significantly lower than the claimed \_\_\_\_\_% discount. Such an event would minimize any discount for marketability.

### b) Use of simple average or median from Restricted Stock Studies

Restricted stock studies are a common source of market data on lack of marketability. One of the original studies, the SEC Institutional Investor Study ("SEC Study"), compared the market prices paid for stock of publicly-traded companies with the prices paid for "restricted" shares of stock in those same companies.

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The restricted shares were generally sold in private placements, or similar transactions, under conditions which prevented them from being re-sold for some period of time (generally two years for the SEC Study).<sup>48</sup> Observed price differences between sales of restricted stocks and their immediately salable equivalents (in those same companies) imply a discount for lack of marketability.

### *SEC Study*

Table 1, Analysis of SEC Institutional Investors Restricted Stock Study, presents detailed data from the SEC Study. According to the source reference, these data were published in the year 1971, and reflect 398 transactions over the years 1966 – 1969.

In regards to Table 1, the following observations are often noted:

- 1) The data indicate that illiquid shares generally sold for less than liquid shares, suggesting an average discount for lack of marketability of 26%;
- 2) The range of variance was significant, however, with groupings ranging from a negative discount of -15% (thus, a premium for lack of marketability), to high-end groupings upwards of an 80% implied discount for lack of marketability; and,
- 3) Greatest weighting of transactions occurred within the “15%” and “25%” implied discount groupings. This suggests a most-common discount for lack of marketability of 20%.<sup>49</sup>

Data in Table 1 are presented in regards to the issue of lack of marketability. However, significant variance in implied discounts for lack of marketability throughout the dataset suggests that factors, other than exclusively marketability, contributed to observed price differences between restricted and unrestricted shares of stock.

### *Management Planning Study*

Table 2, Analysis of MPI Restricted Stock Study, presents detailed data from the Management Planning Study, which also analyzed discounts on sales of restricted stock. According to the source reference, these data are more recent

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<sup>48</sup> The restriction period has generally decreased from two years to one year for similar transactions occurring after the year 1997.

<sup>49</sup>  $(15\% + 25\%) \div 2 = 20\%$ , see note [4] of Table 1.

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than the SEC Study, and reflect 49 transactions over the years 1980 through 1995.

In regards to Table 2, the following observations are noted:

- 1) Figures in the “Average Discounts” column suggest that discounts for lack of marketability *decrease*, as company size (in annual revenues) *increases*;
- 2) However, figures in the “Range of Discounts” columns indicate significant variance within each grouping, with even the smallest companies (under \$10 million in annual revenues) reflecting implied discounts for lack of marketability ranging from a low of 2.8% to a high of 57.6% (see note [2] of Table 2); and,
- 3) While figures in the “Average Discounts” column *decrease* as company size *increases*, low-end figures within the “Range of Discounts” columns for each grouping do not follow this trend. Instead, the lowest discounts observed within the entire range (2.8% and 0.0%) occurred within the smallest and largest company groupings, respectively.

Data in Table 2 are presented in regards to the issue of lack of marketability. However, significant variance in implied discounts for lack of marketability throughout the dataset (including lack of trend for lowest discounts within each grouping) suggests that factors, other than exclusively marketability, contributed to observed price differences between restricted and unrestricted shares of stock.

### *Other factors*

Restricted stock studies have been criticized as being inconsistent with the Fair Market Value standard. Restricted stock sales reportedly reflect transactions among a select group of individuals, with particular motivations for buying/selling under specific conditions. For example, some criticisms argue that discounts on restricted stock and/or private placements represent “compensation” to specific investors who provide guidance and assistance to the company’s management. This suggests that other factors might have affected observed discounts in prices from “marketable” shares of those same stocks.

Other criticisms argue that “blockage” or other “price-pressure” effects might contribute to observed discounts. As one example, suppose a publicly-traded company needed to raise additional capital, but management believed that issuing new public shares would depress the market price (assume a market price of

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\$20/share). A large private placement of restricted shares might then occur at a below-market price (assume a restricted price of \$15/share). This presents the question: Is it appropriate to infer a 25% discount for lack of marketability by comparing the \$15 restricted price to the \$20 market price ( $\$20 - 25\% = \$15$ )?

All else equal, the market price in this example would have decreased below \$20/share if new public shares had been issued. On that basis, the value of the company's comparative "liquid" shares is perhaps less than \$20/share. And therefore, in this example, comparing the \$15 restricted price to the \$20 market price might overstate the implied discount for lack of marketability.

This example also illustrates that restricted stock studies data could reflect transactions of varied buyer/seller motivations. All else equal, the buyer in this example might demand a below-market price to offset risks of investing in a company that was having difficulty raising additional capital. While the seller (the company) might demand restrictions on the new (below-market) shares to protect existing shareholders from a potential drop in stock price.

Observers of this example transaction might then ask themselves...

Were the shares priced below-market because they were restricted?...

Or were the shares restricted because they were priced below-market?

This example illustrates the importance of understanding that observed "discounts" from the market prices of assumed comparative "liquid" shares might include consideration of factors other than exclusively marketability.

### *On excluding other factors*

In regards to excluding other factors, the Bajaj Study<sup>50</sup> explored separation of lack of marketability from other factors believed to affect observed price differences between sales of restricted and unrestricted shares of stock.

The following citation from the Bajaj Study suggests a 7.23% discount for "lack of marketability":

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<sup>50</sup>Bajaj, Denis, Ferris, and Sarin, "Firm Value and Marketability Discounts," Vol. 27, No. 1, Journal of Corporation Law, pp. 89-115, Fall 2001 ("Bajaj Study").

## E. Evaluation and Recommendations

“Therefore, controlling for all other factors influencing private placement discounts, an issuer would have to concede an additional discount of 7.23% simply to compensate the buyer for lack of marketability.”

This statement supports a premise that market data commonly relied upon for estimating discounts for lack of marketability include consideration of other factors.

### *Summary*

An appropriate discount for lack of marketability should not overstate the effects of marketability upon the otherwise determinable pro rata value. The appraiser should use judgment when applying discounts derived from summary median or average data sources to a specific company or subject interest.

#### c) Use of analytical study results without getting behind data

Business valuers will often refer to one or more of the analytical studies and quote certain of the statistics from the studies. For example, a statement may be made that Wruck found a discount for lack of marketability of 17.6%, Hertz & Smith found a discount of 13.5% for lack of liquidity or that Bajaj et al determined that the discount for lack of marketability should be 7.23%. These quotations are then used to build a discount for lack of marketability pertinent to the valuator's assignment or to justify a discount already determined by some other method. Sometimes, one of these figures is simply adopted as representing the appropriate discount for lack of marketability in a given assignment.

It should be remembered that these figures are the result of statistical analysis of a specific data set as chosen by the researcher. The data set in question contains those transactions chosen for one reason or another by the selector and is pertinent to a given time period. Wruck pulled her data from 1979 – 1985, Hertz & Smith studied 1980 – mid-1987 while Bajaj utilized 1990 – 1995. In each case, the sample size was small (128 transactions for Wruck, 106 for Hertz & Smith and 88 for Bajaj). Further, the selection methodology was not well documented and, in each case, relied upon certain assumptions as to registration status and appropriate measurement date. Finally, these studies were all conducted for academic purposes rather than tax purposes investigating various facets of capital formation and shareholder behavior. Although Bajaj eventually extended his study for tax use in his Tax Court testimony it was not originally intended for that purpose.

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A valuator should not use the results from any of the analytical studies without getting behind the data that was used in the various analyses. With respect to Bajaj this is what the Tax Court attempted to do in *McCord*. Rather than accepting the 7.23% discount presented to it in the direct testimony, the Court looked at the data itself and instead determined a discount of 20% for use based on the average discount attributable to Bajaj's middle group of individual transaction results. The Court justified its approach by noting that the transactions in this middle group most closely represented the transaction with which it was confronted in the *McCord* case. In so doing the judge distinguish the present valuation problem from the postulated circumstances attendant to both the highest and the lowest discount groups from the Bajaj study. Whether one accepts the Court's logic in *McCord* or not this is the kind of analysis that needs to be undertaken if one or more of the analytical studies is to be used in framing an opinion on the proper level of marketability discount for a given situation.

### d) Use of study results not supported by market data

It is not uncommon for a valuator to propose a theoretical model as the basis for the determination of a discount for lack of marketability. Having put in chosen parameter values, the model then cranks out a percentage loss in value or a reduced value that can be used to calculate a percentage discount for shortcomings in liquidity or marketability. Although the model may seem conceptually sound in the abstract, there is no attempt to validate the model using actual current market data. For this reason, there is no way for the reviewer to perform a reality check on the model results. Examples of this approach may involve the application of the Quantitative Marketability Discount Model, one of the models based on option theory or one of the analytical approaches based on a limited data set.

The discount for lack of marketability must be firmly based on current market evidence. This point was brought out clearly in the recent summit on DLOM held in San Diego and organized by Judge David Laro and Mel Abraham. No matter how conceptually sound a model may appear to be, unless it can be demonstrated that it produces results that can be verified with market evidence, it remains a theoretical construct that assumes a negotiation pattern between willing buyers and sellers rather than being based on the results of such a pattern. A valuator must remember that a discount for lack of marketability or for anything else is but a step towards arriving at fair market value. Thus, without a verifiable basis in the market, the valuator is asking the audience to take his

## E. Evaluation and Recommendations

result on faith based on what sounds reasonable rather than on what has been empirically demonstrated.

### e) Reliance solely on court decisions

Sometimes a valuator will base a decision as to the choice of marketability discount on previous court decisions. For example, the valuator will review the results of several cases such as *McCord*, *Lappo* and *Peracchio* and then base the choice of discount on the discounts accepted by the court in the reviewed cases. For example, the range of court discounts might have been from 20% to 25% so the valuator chooses 22.5% with the rationale that his valuation subject is similar to the subjects under consideration in the cases cited. Judges are sometimes found to adopt this approach as well. The judge will look at *McCord* with its 20% discount and add a factor of say 3% based on his analysis of the special factors of his case to arrive at a chosen DLOM level of 23%.

It must be remembered that judges are not valuers and are not constrained to the environment in which professional valuers operate. A judge can adopt any approach that is considered useful and can arrive at any result that seems reasonable in his or her view based on all the considerations of the case which often go well beyond the discount for lack of marketability. In addition, a judge will often select one discount over another simply based on the ability or lack thereof that the two sides of the dispute display in arguing their respective cases. The court is a trier of fact and need not, if that is its choice, go beyond what is presented to it. If one side argues persuasively while the other side disappoints the court for one reason or another a discount may emerge without any real justification for why it has been chosen. In fact, the discount selection may not be based on any clear valuation logic at all.

The courts are an excellent source of information when legal precedent is in question but can be a very questionable source when valuation guidance is desired. If the decisions from various court deliberations are to be utilized in the selection of valuation methods or parameters such should be looked at for the underlying reasoning applied and the logic and flow of the judge's thinking not for the results that were finally reached. No two valuation assignments are identical. Therefore, basing one's results on the results of another assignment whether litigated or not is a failure of proper diligence with regard to the assignment presently at hand.

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### 4. Sources available to IRS Valuation Analysts

It has been attempted to gather as many of the underlying studies and research papers referenced in this job aid.

DLOM online sources currently available<sup>51</sup> are listed below. These have limited access and have been purchased for one user—contact information is provided at Engineering BV Resources webpage.

Online databases:

- FMV Restricted Stock Study
- Valuation Advisors Pre-IPO database

#### FMV Restricted Stock Database—Analysis

The FMV Restricted Stock database of transactions is available for purchase, and is utilized by valuers to estimate DLOM on privately-held business interests. An IRS Engineer completed an analysis<sup>52</sup> of the 475 transactions in the FMV Restricted Stock database in 2009. The purpose was a) to analyze the FMV model for determining DLOM on private equity, and b) to determine whether it is possible to develop a statistically valid regression-based model to determine the DLOM. The conclusions drawn are:

- 1) FMV Opinions' model is flawed insofar as explanation of the DLOMs on the restricted stock transactions in their database;
- 2) Valuers cannot confidently rely on FMV's model when determining DLOMs on restricted stocks, much less on interests in private equity; and
- 3) Neither FMV's model nor multivariate regression analysis can be applied to FMV's database to confidently determine the DLOM on private equity.

Please refer to Exhibit A—Review FMV Restricted Stock Model in this job aid for information on the process followed in reaching the conclusions.

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<sup>51</sup> Resources require annual funding to maintain the license. Therefore budgetary limitations may require that these subscriptions be canceled.

<sup>52</sup> February 18, 2009 memo with the subject, "Update: FMV Opinion's Model and Database". Copy is provided as an Exhibit to this job aid.



## ***F. Summary and Conclusions***

This job aid was prepared to assist IRS valuers understand the numerous studies and approaches used by valuation professionals to determine DLOM. We have addressed restricted stock studies, the eldest type of study having been developed in the early 1970's. As the need increased to better quantify DLOM, newer approaches are introduced, such as Liquistat (2007). While many of the newer approaches are not currently used in professional practice, the profession's reliance could change in the future. In total, the job aid has information on 23 different DLOM approaches.

DLOM has been defined as well as the factors that impact DLOM on a specific interest. Sample IDR questions are provided. We have summarized each approach, provided references, identified strengths and weaknesses, how the Tax Court has ruled on the approach, and how prevalent its use in practice. In addition, the job aid offers discussion points to use with Taxpayer appraisers to focus on a specific method.

As is always the key, facts and circumstances surrounding the subject interest are what determine the level of DLOM, if any. DLOM studies, methods and models can be complex, can indicate widely diverse conclusions, and may be appropriate in only certain limited situations. The business valuation profession does not identify acceptable or unacceptable methods for estimating marketability discounts, although some individual practitioners have their own preferences and frequently disagree as to the best approach.

This job aid does not provide guidance on the best DLOM approaches, but is meant to help the reader understand and make an informed decision about DLOM. It is current as of the date of this writing.

For recommendations on the content included in this job aid, employees may contact any of the members of the DLOM Team who were the developers.

## **G. Bibliography**

### **Articles**

Abbott, Ashok. "Empirical Measures of Marketability and Liquidity Discounts, Discount for Lack of Marketability: An Empirical Analysis and DLOM – Concepts and Models." Presentations at Various ASA and NACVA Conferences and on the *BVR Teleconference* of April 26, 2006.

Abbott, Ashok. "New Abbott Analysis Aids Valuers in Assessing Liquidity Discounts." *Business Valuation Update*. Vol. 13, No.11, Nov. 2007.

Amihud, Yakov and Mendelson, Haim "Asset Pricing and the Bid Ask Spread", *Journal of Financial Economics* v17 (December 1986) pp. 223-49.

Bajaj, Mukesh, David J. Denis, Stephen P Ferris & Atula Sarin. *Firm Value and Marketability Discounts*. Journal of Corporation Law. Vol 27, No.1.

Barclay, Michael J., Clifford G. Holderness and Dennis P. Sheehan. "Private Placements and Managerial Entrenchment." *Journal of Corporate Finance* Vol. 13, 2007, pp. 461-484.

*Cost of Flotation of Registered Issues 1971-1972*. Securities and Exchange Commission, 1974.

Dann, L.Y. and H. DeAngelo, 1988, *Corporate Financial Policy and Corporate Control: A Study of Defensive Adjustments in Asset and Ownership Structure*, *Journal of Financial Economics*, 20, 87-127

Edelman, Richard B. and H. Kent Baker, 1993, *The Impact of Company Pre-Listing Attributes on the Market Reaction to NYSE Listings*, *The Financial Review*, Vol. 28, No. 3, August 1993, 431-448

Emory, John D. Sr, F.R. Dengel, III. "Discounts for Lack of Marketability, Emory Pre-IPO Discount Studies 1980-2000 as Adjusted October 10, 2002." *Business Valuation Review*, Vol. 21 No. 4, (December 2002).

Feldman, Stanley Jay. "Revisiting the Liquidity Discount Controversy: Establishing a Plausible Range." *Bentley College and Axiom Valuation Solutions*.

## G. Bibliography

Finnerty, John D. "The Impact of Transfer Restrictions on Stock Prices." *Fordham University, Presentation at the Meeting of the American Finance Association, 2003.*

Frazier, William. "Non-marketable Investment Company Evaluation (NICE)." Valuation Strategies. November/December 2006. Vol 10, No 2.

Garland, Pamela J. and Ashley L. Reilly. "Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study". Available <http://www.willametteinsights.com/03/Autumn2003article2.pdf>, April 30, 2009.

Gomes, Armando and Gordon Phillips. "Why Do Public Firms Issue Private and Public Equity, Convertibles and Debt?" *Presentation at the Research Seminar in Law, Economics, and Organization*, 2005.

Hall, Lance. "New Tactics Required to Prove Discount for Lack of Marketability." *The Value Examiner*, Jan-Feb 2007, pp 36-39.

Hall, Lance S. and Timothy C. Polacek, "Strategies for Obtaining the Largest Valuation Discounts", *Estate Planning*, Vol. 21 No. 1, January/February, 1994.

Hall, Lance, 2004, "The Discount for Lack of Marketability: An Examination of Dr. Bajaj's Approach", *BV Update*, March 2004.

Hertzel, Michael and Richard Smith. "Market Discounts and Shareholder Gains for Placing Equity Privately." *Journal of Finance*, Vol. 48, 1993, pp. 459-485.

Keys, Phyllis and Norris Larrymore. "Integration of Private and Public Offerings." *Presentation at the Conference of the Eastern Finance Association*, 2004.

Koeplin, John, Atulya Sarin and Alan Shapiro, 2000, "The Private Company Discount", *Journal of Applied Corporate Finance*, Volume 12, Number 4, Winter 2000, 94-101.

Longstaff, Francis A. "How much Can Marketability Affect Security Values?" *The Journal of Finance*, December 1995, pp. 1767-74.

Mitchell, Mark and Mary Norwalk, 2008, "Assessing and Monitoring Bajaj", *Business Valuation Review* 27-1, Spring 2008.

## G. Bibliography

Pratt, Shannon , 2004, "The McCord Case and the Bajaj Method", Unpublished Conference Call Pre-Read.

Pratt, Shannon , Chris Mercer, Lance Hall and Rob Oliver, 2004, "DLOM: A Critique of the Bajaj Approach", *BVR Teleconference* April 2004

Pratt, Shannon, 2004, "Defending Discounts for Lack of Marketability", *ACTEC Journal* 2004, 276.

Pratt, Shannon, 2003, "Discounts for Lack of Marketability: Documentation, Critique and Defense", *American Society of Appraisers International Appraisal Conference*, July 2003

Reilly, Robert. "Update on Study of Discount for Lack of Marketability". *American Bankruptcy Institute Journal*, June 2005, Value and Cents

Reilly, Robert and Aaron Rotkowsky, "Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies". Fall 2007, *Tax Lawyer*, Vol 61, No. 1.

Ritter, Jay R. "The Costs of Going Public." *Journal of Financial Economics*, Vol. 19, No. 2 (December 1987), pp. 269-81.

Robak, Espen. "Discounts for Illiquid Shares and Warrants: The LiquiStat Database of Transactions on the Restricted Securities Trading Network." Pluris Valuation Advisors White Paper, January 2007.

Robak, Espen. Liquidity and Levels of Value: A New Theoretical Framework." BV Update, October, 2004.

Paschall, Michael A. *Discounts for Lack of Marketability: A Review of Studies and Factors to be Considered*. Fair Value, September 1994.

Sanger, Gary C. and John J. McConnell, "Stock Exchange Listings, Firm Value, and Security Market Efficiency: The Impact of NASDAQ". *The Journal of Financial and Quantitative Analysis*, Volume 21, Issue 1, March 1986, 1-25

Saunders, Philip. "Marketability Discounts and Risk in Transactions Prior to Initial Public Offerings". *Business Valuation Review*. Volume 19, No. 4, December 2000, pp 186-195.

## G. Bibliography

Seaman, Ronald M. "Minimum Marketability Discounts—2<sup>nd</sup> Edition." *Business Valuation Review*. June 2005, pp. 58-64.

Seaman, Ronald M. "Minimum Marketability Discounts—3<sup>rd</sup> Edition." September 2007. Available at <http://www.dlom-info.com/> August 18, 2009.

Seaman, Ronald M. "Minimum Marketability Discounts—4<sup>th</sup> Edition, A Study of Discounts for Lack of Marketability Based on LEAPS Put Options in November 2008." September 2007. Available at <http://www.dlom-info.com/> August 18, 2009.

Silber, William, 1991, "Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices", *Financial Analysis Journal*, 47, 60-64.

Tabak, David. "A CAPM-Based Approach to Calculating Illiquidity Discounts." Available [http://www.nera.com/Publication.asp?p\\_ID=1152](http://www.nera.com/Publication.asp?p_ID=1152) , November 11, 2002.

Trout, Robert R. "Minimum Marketability Discounts." *Business Valuation Review*. September 2003, pp. 124-126.

Trout, Robert R. "Estimation of the Discount Associated with the Transfer of Restricted Securities." *Taxes*. June, 1997, pp. 381-384.

Wruck, Karen Hopper. "Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings." *Journal of Financial Economics* Vol. 23, 1989, pp. 3-28.

## G. Bibliography

### **Books**

Kasper, Larry J. Business Valuations: Advanced Topics. Greenwood Publishing Group. 1997. Chapter 5 Premium and Discounts.

Mercer, Z. Christopher, Quantifying Marketability Discounts: Developing & Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests, *Peabody Publishing, LP*, 1997.

Mergerstat Review, *FactSet Mergerstat, LLC*, 2006.

Pratt, Shannon P., Robert F. Reilly, Robert P, Schweihs. Valuing A Business. The Analysis and Appraisal of Closely Held Companies, 5<sup>th</sup> ed. New York: McGraw-Hill, 2008.

Pratt, Shannon P. Business Valuation Discounts and Premiums. New York: John Wiley & Sons, Inc., 2001.

### **Websites**

Howard, Frazier, Barker, Elliot, Inc. ([www.hfbe.com](http://www.hfbe.com))

LEAPS ([www.dlom-info.com/articles.html](http://www.dlom-info.com/articles.html))

NERA (National Economic Research Associates) ([www.nera.com](http://www.nera.com))

Options Education (<http://www.optionseducation.org/strategy/collar.jsp> )

Partnership Profiles (aka “Partnership Spectrum”) ([www.partnershipprofiles.com](http://www.partnershipprofiles.com))

**Table 1 Analysis of SEC Institutional Investors Restricted Stock Study**

Reference – Institutional Investor Study (“SEC Study”) data<sup>53</sup>

Discounts on Purchase Price of Restricted Common Stock

Average Discount	Total Transactions	<=====Range of Discounts by Respective Groupings <sup>54</sup> =====>													
		-15% to 0% discount		0% to 10%		10% to 20%		20% to 30%		30% to 40%		40% to 50%		50% to 80%	
		Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total	Trans. in this Grouping	As a Percent of Total
26%	398	26	6.5%	67	16.8%	78	19.6%	77	19.3%	67	16.8%	35	8.8%	48	12.1%

Weighted Average “Discount”<sup>55</sup>

	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting	Grouping Discount	Respective Weighting
	-7.5%	6.5%	5.0%	16.8%	15.0%	19.6%	25.0%	19.3%	35.0%	16.8%	45.0%	8.8%	65.0%	12.1%		
26%	Weighted Average Discount															
	Greatest weighting within these two groupings <sup>56</sup>															

<sup>53</sup> Source: Quantifying Marketability Discounts, by Z. Christopher Mercer, ASA, CFA, *Peabody Publishing, LP*, 1997, Exhibit 2-1, page 70.

<sup>54</sup> Data range includes a low of -15% (a negative discount), to a high of 80% discount.

<sup>55</sup> Calculated result equals the reported 26% “Average Discount”.

<sup>56</sup> Greatest weighting of transactions occurs within the 15% and 25% implied discount groupings, suggesting a most common discount of 20%.

**Table 2 Analysis of MPI Restricted Stock Study**

Reference—Management Planning Study data<sup>57</sup>

Analysis of Restricted Stock Discounts by Revenue Size

Revenues	Number of Observations	Average Revenues (\$ millions)	Average Discounts	Range of Discounts	
				Low	High
Under \$10 million <sup>58</sup>	14	\$6.6	32.9%	2.8%	57.6%
\$10 - \$30 million	11	\$22.5	30.8%	15.3%	49.8%
\$30 - \$50 million	10	\$33.5	25.2%	5.2%	46.3%
\$50 - \$100 million	8	\$63.5	19.4%	11.6%	29.3%
Over \$100 million (adjusted)*	4	\$224.9	14.9%	0.0%	24.1%
Overall sample averages		\$47.5	27.7%	0.0%	57.6%
Totals	47				
*Over \$100 million (actual calculation)	2	\$187.1	25.1%	0.0%	46.5%
Totals	49				

Excludes Sudbury Holdings, Inc., whose private placement consisted of 125% of the pre-transaction shares outstanding.  
Excludes Starrett Housing Corp., which is one of the five most thinly traded companies in the sample.

<sup>57</sup> Source: Quantifying Marketability Discounts, by Z. Christopher Mercer, ASA, CFA, *Peabody Publishing, LP*, 1997, Figure 12-1, page 346.

<sup>58</sup> Discounts for the smallest companies occurred over a range of [2.8% - 57.6%].



## ***Exhibit A—Review FMV Restricted Stock Model***



DEPARTMENT OF THE TREASURY  
INTERNAL REVENUE SERVICE  
WASHINGTON, D.C. 20224

DATE: February 18, 2009

TO: Engineering Territory Manager, Field Specialists West  
National Business Valuation Issue Coordinator

THRU: Manager, Field Specialists Team

FROM: Engineer, Field Specialists Team

SUBJECT: Update: FMV Opinions' Model and Database

This memorandum is a summary of the results of my two-part assignment 1) To analyze FMV Opinions' (FMV) model for determining the discount for lack of marketability (DLOM) on private equity, and 2) To determine whether it is possible to use FMV's 475-transaction database to develop a statistically valid regression-based model to determine this discount.

It begins with a description and critique of FMV's model, continues with a discussion of multiple regression, and concludes with a statement as to whether multiple regression can be applied to FMV's database in order to confidently ascertain the DLOM on private equity.

The bottom line of my inquiry is threefold:

- 1) FMV Opinions' model is flawed insofar as explanation of the DLOMs on the restricted stock transactions in their database;
- 2) Valuers cannot confidently rely on FMV's model when determining DLOMs on restricted stocks, much less on interests in private equity; and
- 3) Neither FMV's model nor multivariate regression analysis can be applied to FMV's database to confidently determine the DLOM on private equity.

### Description of FMV Opinions' Model

My understanding of FMV's model is based on two articles by Espen Robak, CFA (Robak): "Marketability discounts: Using four measures of risk and adjusting for relative liquidity" and "FMV introduces detailed Restricted Stock Study."<sup>59</sup> These are not included, but are available upon request.

Following are four quotes from these two articles that explain FMV's thinking and methodology. The first three are from "Marketability discounts: Using four measures of risk and adjusting for relative liquidity":

- . . . there are inherent and significant differences between the liquidity of restricted stock and the liquidity of . . . private equity . . .
- We determine the "restricted stock equivalent" discount, given the financial characteristics of the company . . . We determine the incremental discount for the privately held company . . . based on a review of the most illiquid (i.e., large-block) restricted stock issues.
- We must adjust for the fact that private equity is more illiquid than the *typical* block of restricted stock in a public company.
- In terms of liquidity, large blocks are essentially like private equity.

The last is from "FMV introduces detailed Restricted Stock Study":

- . . . the discounts for restricted stock with longer-than-average holding periods are particularly applicable to privately held securities. The only question is "How do we isolate the transactions with longer holding periods from the rest of the sample? The answer is: "by looking at transactions in large blocks until 1997."

**Attachment 1** is a 1-page summary of FMV's two-step methodology as it was explained in Robak's article, "FMV introduces detailed Restricted Stock Study." With rounding, the 50% DLOM in cell G59 is the same as the final indicated DLOM in Robak's article.

In short, FMV's model is based on the proposition that the DLOM on private equity is a function of firm characteristics (rows 9-31 in Attachment 1), and the incremental difference between discounts on the smaller and larger blocks of restricted stock (rows 36-50). Certain related cells have been identified by shading to enable the reader to more easily follow the less obvious calculations.

### Critique of FMV Opinions' Model

Here are three major problems with FMV Opinions' model, and then eight less consequential questions and comments.

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<sup>59</sup> The second article appeared in Shannon Pratt's *Business Valuation Update*, November 2001, pp. 1-3. I don't know where the first one appeared.

### Major Problems

- 1) FMV has not shown that block size is a statistically reliable proxy for quantifying the supposed difference in liquidity between restricted stocks and private equity.
- 2) The first step in FMV's analysis (rows 9-31 in my Attachment 1) is intended to account only for the financial risk characteristics of the subject entity, but it also reflects block size. The second step (rows 36-50) is intended to segregate and quantify the impact of block size, but it is also influenced by the same financial risk characteristics in the first step. While meant to be separate steps, the two are interrelated and, therefore, corrupted by each other.
- 3) According to Robak's article, the first step in FMV's analysis is to "determine the 'restricted stock equivalent' discount given the financial characteristics of the company . . ." The tables in Robak's article – and on lines 11-14 in my Attachment 1 – show four such firm characteristics. The discounts in the 475-transaction survey given me are inconsistent with this purpose because they are a function of both firm characteristics (such as Z-score<sup>60</sup> and Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA)), and issue characteristics (such as per-share offering amount, and block size as a percent of total shares.) They are the discounts on particular offerings, and not indicators of marketability or lack thereof.

### Other Questions and Comments

- 4) There is not a statistically significant relationship between discounts to market value and the four risk characteristics in Robak's article. I regressed the discounts in the database against market values, market-to-book (MTB) ratios, net profit margins, and dividend yields, and found a 0.0238 R-square<sup>61</sup>, coefficients' signs contrary to economic theory, and an F-statistic low enough that the regression equation could not be relied on even at the 10% level of significance<sup>62</sup>.
- 5) Selection of how many and which particular firm characteristics to include in the model, appears subjective, and has not been explained. Robak has only noted "we use different indicators in different situations."
- 6) FMV has not shown why survey quintiles are the best way – or even a reliable way – of extrapolating survey results to a subject outside the

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<sup>60</sup> Developed in 1968 by Edward I. Altman, Ph.D., Z-score is a formula for predicting bankruptcy and a numerical indicator of a firm's financial health.

<sup>61</sup> R-square, or coefficient of determination, is a measure of the degree of association between the response variable (the discount) and independent variables, as a whole. By comparison, the most often quoted minimum acceptable R-square is 0.80.

<sup>62</sup> The *F*-test is a comparison of the *F*-statistic and critical values of *F* at various significance levels. In this case, 1.293 vs 1.972 *F*-critical at the 10% significance level means one couldn't even be 90% certain that the regression equation explains observed discounts.

survey. Why weren't quartiles or deciles or multiple regression used instead?

- 7) The restricted stock equivalent on line 15 in Attachment 1 is dependent on whether the analyst chooses to use quintiles or some other measure. One would get a different result using, say, quartiles or deciles.
- 8) The example in Robak's article - and summarized in my Attachment 1 - did not include the characteristic thought by them<sup>63</sup> to be most correlative with marketability: stock price volatility.
- 9) FMV Opinions did not explain why they used medians instead of means.
- 10) FMV Opinions did not explain why they used "additive" and "multiplicative" calculations to determine the Private Company Increment in step 2. Why not just one or the other?
- 11) FMV Opinions didn't explain how they selected the matching transactions in rows 17-23. It's presumed that these were the only transactions in the same factor quintiles.

The last eight of these factors question FMV's model. The first three disprove it.

### Regression Analysis

Robak refers to regression analysis in his article, "Marketability discounts: Using four measures of risk and adjusting for relative liquidity", but he doesn't explain why FMV's model doesn't use it, or go on to discuss it in either of his articles.

I reviewed FMV's 475-transaction spreadsheet<sup>64</sup> and found two combinations of firm and issue characteristics that, arguably, *do* result in regression models that explain the observed discounts in these transactions. These are in my Attachments 2 and 3.

**Attachment 2** shows that, collectively, stock price volatility, per-share offering price, offering as a fraction of total shares outstanding, and offering dollar amount, could be used to make predictions of discounts for restricted stock transactions outside the database. The 22.20 *F*-statistic, significant even at the 1% level (22.20>3.36), indicates that one could be 99% confident that this combination is related to the discount in a restricted stock transaction.

In regard to interests in private equity - which is more often the subject of IRS valuations - this model is lacking because -

- 1) It could not be used to determine lack-of-marketability discounts on interests in private companies; and

<sup>63</sup> BVR April 2006 Telephone Conference: "Discounts for Lack of Marketability", Page 2.

<sup>64</sup> Of the 563 trillion combinations of firm and issue characteristics in the 53-column, 49-"driver" spreadsheet, I tested approximately thirty of the most promising combinations of characteristics.

- 2) According to a number of authors (most of whom have Ph.D.'s in economics)<sup>65</sup>, it does not control for (isolate and quantify) the non-marketability determinants of observed discounts. Unlike the traditional Restricted Stock Approach, these two studies suggested that only a portion of the overall discount represents a lack-of-marketability.

With the second limitation of Attachment 2 in mind, I tested other combinations of characteristics to see if there might be a combination which *does* control for non-marketability factors. Of the combinations of characteristics I looked at<sup>66</sup>, the model in **Attachment 3** comes closest to allowing for determination of the DLOM on interests in private equity.

This model is adequate in that -

- The *F* test indicates that, collectively, this combination of characteristics is adequate for prediction of the DLOM on restricted stock.;
- All four coefficients are consistent with economic theory; and
- By differentiating between stocks with registration rights and stocks without registration rights, the “Registered vs Restricted” term<sup>67</sup> serves the purpose of quantifying the impact of marketability.

Unfortunately, this model, too, is lacking. While it could have been argued that the DLOM on restricted stock is 2.42%<sup>68</sup> were it not for the registration indicator's statistically unacceptable 0.188 P-value, the registration indicator's high P-value (greater than 0.1) invalidates use of this model for determining DLOMs.

#### Summary – Attachments 2 and 3

While there are at least two combinations of firm and issue characteristics in FMV's database that adequately explain the discounts in it, neither can be used to predict the lack-of-marketability discount on an interest in a private company.

The regression models in Attachments 2 and 3 both fail because of the difference between private equity and the restricted stocks in FMV's database. The model in Attachment 2 also fails because it does not include “registration indicator”, arguably the only variable that measures marketability. The model in Attachment 3 also fails because the P-value of the “registration indicator” renders this model less than a reliable predictor.

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<sup>65</sup> Bajaj, et al “Firm Value and Marketability Discounts”, 27 J. Corp. L. 89, 98 (2001) and Hertz and Smith's “Market Discounts and Share holder Gains for Placing Equity Privately”, 48 J. Fin. 459 (1993).

<sup>66</sup> This included holding period, offering price, block size, stock price volatility, z-score, retained earnings, market value, revenues, and whether or not registered.

<sup>67</sup> Also called the “registration indicator.”

<sup>68</sup> The coefficient of “r.”

## Conclusion

FMV Opinions' 2-part model is based on the proposition that the DLOM on private equity is the sum of –

- The relationship between the financial risk characteristics of, and the discounts on restricted stocks; and
- The difference between the discounts on smaller and larger blocks of stock.

Three of the more important problems with FMV Opinions' model are:

- 1) FMV has not shown that block size is a statistically reliable proxy for the supposed difference in liquidity between restricted stocks and private equity;
- 2) The two steps in FMV Opinions' model are interrelated and, therefore, corrupted in that they are both influenced by firm characteristics and by block size; and
- 3) The discounts in FMV's database are the discounts on particular offerings, comprised of marketability and non-marketability factors, and not necessarily indicators of marketability or the lack thereof.

Applying multivariate regression analysis to FMV's database, I found two combinations of four firm and issue characteristics which could be used, with some degree of statistical certainty, to explain the discounts in FMV's study.<sup>69</sup> The difference between them is that the model in Attachment 3 includes "registration indicator", the variable certain authors<sup>70</sup> believe is the only one measuring marketability.

Whether looking at the model in my Attachment 2 or the model in my Attachment 3, or whether one agrees with the proposition that the registration indicator, alone, measures marketability, the fact is:

- 1) FMV Opinions' model cannot be used to reliably predict lack-of-marketability discounts on restricted stock transactions, much less interests in private equity; and
- 2) No one – FMV using their methodology, or someone using regression analysis – could use FMV's database to confidently predict lack-of-marketability discounts on either restricted stock transactions or interests in private equity.

If you have any questions, please contact me.

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<sup>69</sup> The 0.17 R-square notwithstanding. It is less than problematic because of the large number of data points.

<sup>70</sup> Bajaj et al and Hertz and Smith.

# ATTACHMENT 1

## FMV OPINIONS' 2 STEP MODEL PER ESPEN ROBAK ARTICLE

**B                      C                      D                      E                      F                      G**

### Step 1 - Restricted Stock Equivalent Basis

9	Dimension	Variable	Subject Company	Median Discount -	
10			Value Quintile	Subject Company's Quintile	
11	Size	Market Value	\$15,000,000	5	37.3%
12	Risk	Market-to-Book Ratio	8.5X	2	29.5%
13	Profitability	Net Profit Margin	9%	1	15.5%
14	Distributions	Dividend Yield	5%	1	13.1%
15	Average = Indicated Restricted Stock Equivalent, Part A				24%

17	Matching Companies	Quintiles				Private Placement DLOM
18		Market Value	Market-to-Book Ratio	Net Profit Margin	Dividend Yield	
19	Company A	5 = 5 <sup>th</sup> quintile	2 = 2 <sup>nd</sup> quintile	1 = 1 <sup>st</sup> quintile	1 = 1 <sup>st</sup> quintile	29.3%
20	Company B	5	2	1	1	42.6%
21	Company C	5	2	1	1	18.9%
22	etc.	5	2	1	1	
23	Reconciled/Indicated Restricted Stock Equivalent - Part B					32%

27	Restricted Stock Equivalent Discount Indicated by Quintile Medians, Part A	24%
28	Restricted Stock Equivalent Discount Indicated by Matching Companies, Part B	32%
29	Restricted Stock Equivalent Discount, Average of Parts A and B	
30	(The discount at which the company would issue restricted stock, if it were a public company with the same financial risk characteristics)	28%

### Step 2 - Private Company Increment

36	Percent of Shares Placed	Median Discount	Additive	Multiplicative	Indicated DLOM
37	Small Block Sample	27.2%	NA	NA	NA
38	More than 25%	35.3%	8.1%	1.3	35%
39	More than 30%	47.7%	20.5%	1.8	49%
40	More than 35%	59.0%	31.8%	2.2	60%

43	Percent of Shares Placed	Additional Private Company Adjustment ( Adjustment for the fact that private equity is more illiquid than the typical block of restricted stock in a public company. Based on presumption that large blocks are essentially like private equity.)
44	Small Block Sample	NA
45	More than 25%	8%
46	More than 30%	22%
47	More than 35%	33%

### Reconciliation - Selected Discount for Lack of Marketability - Private Equity

55	Step 1 Restricted Stock Equivalent Discount	Private Company Increment			Indicated DLOM (Step1 + Step 2)
56		Median Discount, Small Block Sample	Step 2 Indicated DLOM	Step 2 Difference	
57	28%	27.2%	49%	22% = 49% - 27.2%	50% = 28% + 22%

## ATTACHMENT 2

Following are the regression equation and a tabular presentation of model validity statistics regarding a regression-based model that, arguably, explains the discounts in FMV's 475-transaction database.

The relatively low R-square term notwithstanding, this model is arguably adequate because -

- Compared to the 22.2 F-statistic, the critical values of *F* indicate that, *collectively*, these four variables explain the observed discounts;
- All of the coefficients have signs that are consistent with economic theory; and
- The P-values are such that there is a 95% probability that there is a strong relationship with the discount for three of them, and a 90% probability that there is a strong relationship for the fourth.

$$d = 0.1505 + 0.0490v - 0.0016s + 0.3948p - 0.000000000636a$$

where d = discount

v = stock price volatility

s = offering price per share

p = offering as a % of pre-offering shares outstanding

a = offering amount

R- Square Term *		0.1612
F- Statistic		22.20
F-critical @ ____ Significance Level	10%	1.96
	5%	2.39
	2.5%	2.21
	1%	3.36
P-Values	Volatility	8.81E-11
	Offering \$/Share	0.0357
	Block Size - % of Total Outstanding	0.0001
	Offering Amount	0.0601

\* Note: While 80% is the most widely quoted minimum R-square, the 16% R-square in this case is less important than the *F*-test and the P-values.



## ATTACHMENT 3

# A Regression-based Model That 1) Controls for the Non-marketability Components in Discounts to Market Price in Restricted Stock Transactions, and 2) Isolates the Lack-of-Marketability Component

## Regression Equation

$$d = 0.1143 + 0.0878v - 0.0020s + 0.4186p - 0.0242r$$

where d = discount

v = stock price volatility

s = offering price per share

p = offering as a % of pre-offering shares outstanding

r = registration indicator (1 if the stock was issued with registration rights, and 0 if the stock was NOT issued with registration rights)

## Model Validity Statistics

R- square Term		0.1703
F- Statistic		16.98
F-critical @ ____ Significance Level	10%	1.96
	5%	2.40
	2.5%	2.82
	1%	3.38
P-Values	Registered vs Restricted	1.88E-01
	Offering \$/Share	3.45E-04
	Block Size - % of Total Outstanding	7.50E-05
	Volatility	4.38E-07

## Comments

- These four variables explain 17% of the variation in the discounts in this sample. While 80% is the most widely quoted minimum, the 17% R-square in this case is less important than the *F*-test and the P-values.
- The 16.98 *F*-statistic, significant even at the 1% level ( $16.98 \gg 3.38$ ), indicates that *collectively*, one can be 99% confident that these four characteristics influence discounts.
- The P-values of three of these are far less than 1%.
- The reason why this model is arguably better than the model in Attachment 2, but still inadequate for determining the DLOM on interests in private equity, is discussed on Page 5.

## Exhibit B—Pre-IPO Studies

**Willamette Management Associates<sup>71</sup> (WMA) Original:** A series of studies on the prices of private stock transactions relative to those of public offerings of stock of the same companies. The studies covered the years 1975 through 1997.

Time Period	Number of Companies	Number of Transactions	Standard Mean Discount	Trimmed Mean Discount(*)	Median Discount	Standard Deviation
1975-78	17	31	34.0	43.3	52.5	58.6
1979	9	17	55.6	56.8	62.7	30.2
1980-82	58	113	48.0	51.9	56.5	29.8
1983	85	214	50.1	55.2	60.7	34.7
1984	20	33	43.2	52.9	73.1	63.9
1985	18	25	41.3	47.3	42.6	43.5
1986	47	74	38.5	44.7	47.4	44.2
1987	25	40	36.9	44.9	43.8	49.9
1988	13	19	41.5	42.5	51.8	29.5
1989	9	19	47.3	46.9	50.3	18.6
1990	17	23	30.5	33.0	48.5	42.7
1991	27	34	24.2	28.9	31.8	37.7
1992	36	75	41.9	47.0	51.7	42.6
1993	51	110	46.9	49.9	53.3	33.9
1994	31	48	31.9	38.4	43.0	49.6
1995	42	66	32.2	47.4	58.7	76.4
1996	17	22	31.5	34.5	44.3	45.4
1997	34	44	28.4	30.5	35.2	46.7

(\*) Excludes the highest and lowest deciles of indicated discount

**Willamette Management Associates (WMA) Subsequent:** Over the last several years, *Willamette Management Associates* conducted 12 studies on the prices of private stock transactions relative to those of subsequent offerings of stock of the same companies. The 12 studies covered the years 1975 through 1992. Each private transaction was compared with the subsequent public offering price. In one of the studies, Willamette checked trading prices six months after the initial public offering to see whether the IPO prices were upwardly or downwardly biased compared to a more seasoned market price. While after six months some prices increased and some decreased, the average change from the IPO price was insignificant.

<sup>71</sup> Willamette's studies are unpublished. However, the summary tables are presented in Business Valuation Discounts and Premiums, Chapter 5, by Shannon Pratt (New York: John Wiley & Sons, Inc., p. 85).

The findings of the Willamette studies are summarized in the following table.

STUDY PERIOD	NUMBER OF IPO PROSPECTUSES REVIEWED	NUMBER OF QUALIFYING TRANSACTION	DISCOUNT MEAN	DISCOUNT MEAN
1991-1993	443	54	45%	44%
1990-1992	266	35	42%	40%
1989-1990	157	23	45%	40%
1987-1989	98	27	45%	45%
1985-1986	130	21	43%	45%
1980-1981	97	13	60%	55%
All Studies	1,191	173	N/A	N/A

Of the twelve (12) studies conducted by Willamette, each compared pre-IPO transactions to the IPO price in estimating the marketability discount. One study went one-step further by observing price changes in the publicly-traded stocks six-months subsequent to the IPO. Although this one study found that after six months the change in the subject stock prices from the IPO prices were insignificant; a six month time period allows for market and company specific conditions to change. It should be noted that no conclusions regarding marketability discounts were based on the post-IPO pricing.

**Robert W. Baird & Company Studies (Emory) Original:** John D. Emory of Robert W. Baird & Company conducted pre-IPO studies. The studies covered various time periods from 1981 through 1997. The basic methodologies for the eight studies were identical. The population of companies in each study consisted of initial public offerings during the respective period in which Baird & Company either participated or received prospectuses. The prospectuses of these over 2,200 offerings were analyzed to determine the relationship between (1) the price at which the stock was initially offered to the public and (2) the price at which the latest private transaction occurred up to five months prior to the IPO.

In 2002, John Emory updated his studies in an article titled, "Discounts for Lack of Marketability, Emory Pre-IPO Discount Studies 1980-2000 as

Adjusted October 10, 2002.”<sup>72</sup> This source also has “expanded” and “Dot-Com” data for 1997-2000.

A summary of Emory’s Pre-IPO studies is shown below:

<b>STUDY PERIOD</b>	<b>NUMBER OF IPO PROSPECTUSES REVIEWED</b>	<b>NUMBER OF QUALIFYING TRANSACTION</b>	<b>DISCOUNT MEAN</b>	<b>DISCOUNT MEDIAN</b>
<b>1997-2000</b>	1847	36	48	44
<b>1995 - 1997</b>	732	91	43	42
<b>1994 - 1995</b>	318	46	45	45
<b>1991- 1993</b>	443	54	45	44
<b>1990 - 1992</b>	266	35	42	40
<b>1989 - 1990</b>	157	23	45	40
<b>1987 - 1989</b>	98	27	45	45
<b>1985 - 1986</b>	130	21	43	43
<b>1980 - 1981</b>	97	13	60	66
<b>All 9 studies</b>	4,088	346	46 %	45 %

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<sup>72</sup> *Business Valuation Review*, Vol. 21 No. 4 (December, 2002).

## ***Exhibit C–Analytical Approach Revisited***

### **Other Reviewed Studies**

An introduction to the analytical approach to estimating the discount for lack of marketability was provided in the main body of this job aid. This Exhibit provides summaries of six additional studies that utilize an analytical approach. These summaries are included herein for the interested reader who may want a further background in this growing area of analysis into understanding the mechanics of corporate capital formation.

The authors and dates of the included studies are:

John D. Finnerty – 2003  
Michael J. Barclay, Clifford G. Holderness and Dennis P. Sheehan – 2007  
Phyllis Keys and Norris Larrymore – 2004  
Armando Gomes and Gordon Phillips – 2005  
Stanley Jay Feldman – Undated  
Espen Robak – 2007

The study summaries are followed by a commentary on the strength and weaknesses of these types of studies. However, these studies are basically academic in nature and are not traditionally used by the valuation community to set discount numbers. Nor have they been vetted in any meaningful way by the courts.

### **John D. Finnerty, 2003, *The Impact of Transfer Restrictions on Stock Prices*, Fordham University, Presentation at the Meeting of the American Finance Association**

Finnerty was interested in the impact of transfer restrictions on stock prices and set about to investigate the factors responsible for the discount when unregistered shares of common stock are privately placed. He analyzed a sample of 101 private placements of transfer-restricted stock all of which involved unregistered shares. Finnerty postulated that the following factors influenced the size of required placement discounts:

- Volatility of the publicly traded stock of the entity
- Length of the restriction period
- Risk-free interest rate
- Dividend yield
- Information requirements

#### -Ownership concentration effects

His premise was that, beyond marketability concerns, a discount compensates an investor for the due diligence and monitoring costs required by the investment, the effects due to ownership structure change and the implied certification effect of having a private investor choose the firm for investment. Finnerty asked the question that, if the proper discount is as large as is implied by the so-called benchmark studies, why don't arbitrageurs enter the market, use hedging opportunities and capture a guaranteed profit? He believed that the existence of the equity derivative markets has acted to significantly reduce the discounts that might have existed in earlier years.

Finnerty's analysis covered private placements during the period January 1, 1991 through February 3, 1997 and involved 101 placements of which 77 were either traded on NASDAQ or over-the-counter. His measurement dates were 10 trading days prior to the announcement date and the day prior. The average discounts calculated were 20.13% for the day prior measurement and 18.41% for the 10 day prior measurement. The respective median discounts were 15.50% and 16.74%.

A regression analysis was performed which indicated that the most significant variables influencing total discounts are the volatility of the stock and the length of the restriction period. Finnerty concludes based on this analysis that a range of discounts from 25% to 35% is proper for moderate volatility stocks (30% to 120%) but that this range is too high for low volatility stocks (<30%). For low volatility stocks (20% to 30%), he found the following ranges to apply:

- Dividend paying: 11.5% to 16.0%
- Non-Dividend paying: 15.8% to 20.1%

These ranges apply assuming a 2 year holding period; if a 1 year holding period is assumed the discounts are reduced by about 50%.

Finnerty found no significant relationship between the discount amount and the regression variable representing ownership concentration. This is consistent with the findings of Hertzels & Smith but contrary to the findings of Wruck.

**Michael J. Barclay, Clifford G. Holderness and Dennis P. Sheehan, 2007, *Private Placements and Managerial Entrenchment*, Journal of Corporate Finance 13, 461-484,**

These researchers studied the question of private placements and managerial entrenchment. Their premise was that many private placements are made to facilitate management entrenchments rather than for benefiting the firm from monitoring and certification services provided by the involved investors. The

thesis of using private placements at discounts to entice investment by allies to management was earlier advanced by Dann & DeAngelo and by Wruck. Whereas the monitoring and certification effects hypotheses are favorable to existing shareholders, the management entrenchment hypothesis is not.

Barclay *et al* believe that although private placements result in short-term positive effects on firm value, the long-term effect is negative and leads to firm value declines. Most private placement investors are seen as passive with only about 12% estimated to be active participants in firm governance and management persuasion efforts.

These researchers analyzed data from the years 1979 through 1997 and considered only those placements involving more than 5% of outstanding common stock. Some 594 placements were identified with the investors classified as 12% active, 5% managerial and 83% passive. The average discount found was 18.7% from traded value with a median discount of 17.4%. The various kinds of investors had discount ranges as follows:

- Active: 1.8% average and 7.5% median
- Managerial: 24.2% average and 18.2% median
- Passive: 20.8% average and 19.5% median

Barclay *et al* found that registration status is not a substantial factor in the discount amounts.

Private placements reduce the chances of a downstream acquisition or merger by about 50%. There is no distinction in this statistic based on the kinds of investors involved.

Trades of significant sized blocks of stock in the public market were also studied with 204 data points considered. Investors acquiring interests in such trades were more likely to take an active role in the firm and these investments were more likely to lead to a downstream acquisition. These types of trades were also more likely to be opposed by management since it has no control over who the buyers might be.

Per Barclay *et al*, the cost of private placements is about two times the cost of a seasoned (non-IPO) equity offering when discounts and out-of-pocket costs are considered. This is not the most cost effective way to raise capital and gives support to the thesis that management is proceeding in this way for its own protection and betterment. Passive investors are considered the norm for private placements per Barclay since such represent over 80% of the investors in the transactions study. Thus, they believe that the discounts required by passive investors are most representative of reality.

**Phyllis Keys and Norris Larrymore, 2004, *Integration of Private and Public Offerings*, Presentation at the Conference of the Eastern Finance Association**

Keys and Larrymore studied the question of integrating public and private share offerings to receive the best overall effect with regard to the raising of capital. They note that private placements relate to a less restrictive offering process but also involve asymmetry of information between investors and management. Public offerings require more complete information to be available to investors but also require more disclosure and a less flexible issuing procedure. SEC Rule 155 instituted in March 2001 grants issuers more flexibility to offer securities privately following a public offering and vice versa. In general, the market responds positively to private offerings and negatively to seasoned public offerings.

Keys and Larrymore studies a total of about 1,020 equity offerings for 2000 and 2002 (before Rule 155 and after Rule 155). Some 710 of these were public offerings and about 310 were private offerings. They found that there is a significant difference in the two day market reaction to private as compared to public offerings both before and after Rule 155. There is, however, no significant long-term (120 day) difference.

This conclusion relates to the work of Wruck, Hertzelt & Smith and Bajaj *et al* as these researchers made their measurements very close to the announcement date and did not include a longer-term perspective. Analyzing the Keys result would indicate that much of the discount found for private offerings may be due to a short term bounce effect that gradually disappears.

**Armando Gomes and Gordon Phillips, 2005, *Why Do Public Firms Issue Private and Public Equity, Convertibles and Debt?* Presentation at the Research Seminar in Law, Economics and Organization**

Gomes and Phillips (G & P) were interested in the question of why public firms issue various kinds of equity, convertibles and debt using both public and private issuance modes. They believe that asymmetric information effects and moral hazard problems play a large role in the public versus private market choice and the security type choice. In the private market, they find that firms with high measures of asymmetric information are more likely to issue equity where the public market would prefer debt. Firms with high risk, low profitability and good investment opportunities for acquired capital are more likely to issue equity and/or convertibles publicly than to do so privately. Private securities give investors more incentives to produce information and also to monitor the firm.



Gomes and Phillips studied 13,000+ security issuances with more than half in the private market where public companies were the issuers. They used the accuracy of analyst earnings predictions and the dispersion of analyst errors as a proxy for asymmetric information effects for private equity and convertibles and for public debt. They hypothesized that public debt and all kinds of private offerings serve a “disciplining” function for management due to increased monitoring.

Per Gomes and Phillips, the level of abnormal returns (returns above average) should be positively correlated with the degree of information asymmetry for private offerings; the more information sensitive the security's value, the stronger the correlation should be. As risk rises in a firm's operations it will tend to move from public debt to private debt and from private debt to convertibles to raise capital. Private equity is issued when risk levels rise to the point where it is the last alternative. Agency problems between management and shareholders increase the need for the use of debt or private equity placements to prevent management distortions and abuses.

The G & P database consisted of 13,282 issuances during the period January 1995 through December 2003 and involved 4,137 different firms. No secondary offerings or short-term offerings were included. It was determined that the market reacts most favorably to private equity issuances and least favorably to public equity issuances. Once again, this finding indicates that the discount found with regard to private placements as opposed to public traded prices may result from a short-term bounce effect that then disappears with time. Marketability may not be a significant concern in the overall picture.

**Stanley Jay Feldman, *Revisiting the Liquidity Discount Controversy: Establishing a Plausible Range*, Bentley College and Axiom Valuation Solutions**

Feldman focused his attention on the liquidity discount controversy and worked to establish a plausible range for such discounts. He found the following in summary:

- Minority privately held C Corp shares have a liquidity discount in the area of 14%
- Minority S Corp shares are less liquid than C Corp shares
- Control shares of a C Corp have discounts in the area of 20%
- Discounts >30% for any share blocks are not supported by research

Feldman believes that liquidity may be as important as risk in determining stock returns. Investors demand discounts for lack of liquidity as compensation for their higher costs of trading. Liquidity can be measured by the increase in share price if an OTC firm gets listed on the New York Stock Exchange (NYSE) but such a

rise in stock price can also be due to other factors such as information signaling (the willingness to disclose) and certification by seasoned investor buy-ins. Feldman examines abnormal returns to control for the effects of overall market movements. He reviews the data from a 1966-1970 study by Sanger and McConnell on movements from OTC trading to the NYSE since in this timeframe the NASDAQ did not yet exist. In this study an implied liquidity discount of 20% was found based on an overall set of cumulative abnormal returns of 25.68%. Of these abnormal returns analysis indicates that about 14% are due to lack of liquidity and about 11% are due to information signaling. The liquidity effect is estimated based on a 1982-1989 study by Edelman and Baker on stocks moving from the NASDAQ to the NYSE.

The 14% level of abnormal returns converts to a straight liquidity discount of about 12%. This is seen as being a floor for liquidity discount since there are shares that need to be analyzed that are not traded on any exchange. For these shares, Feldman believes that a 3% additional abnormal return amount is appropriate implying an added discount of 2.5% and resulting in an overall liquidity discount of 14.5% for these kinds of shares.

Feldman notes that this is close to the discount derived by H & S for lack of marketability (13.5%) but is much lower than the discounts implied by the pre-IPO and Restricted Stock Studies of >35%. He believes that there are a myriad of problems with these latter studies that make them very unreliable including high ranges among results and wide coefficients of dispersion that make the use of measures of central tendencies (averages and medians) generally unreasonable.

Feldman cites a study by Koeplin et al that estimates marketability discounts for control interests in the range of 18% to 30% with a reasonable estimate of the average at 20%. He believes that an additional increment of <5% should be added on for S Corp shares. Finally, he concludes that for pure liquidity estimation purposes, control blocks are less liquid than minority blocks with the difference being about 5.5% (20% for control versus 14.5% for minority). However, he admits that minority blocks might give rise to higher overall discounts due to the effects of information signaling and certification.

**Espen Robak, 2007, *Discounts for Illiquid Shares and Warrants: The LiquiStat Database of Transactions on the Restricted Securities Trading Network*, Pluris Valuation Advisors White Paper, January 2007**

Robak postulates that it is not possible to precisely sort out the effect of illiquidity from all of the factors contributing to total discounts in private placement transactions, be they of registered shares or unregistered shares. Instead, he analyzes a specially constructed database referred to as LiquiStat which

incorporates data on investor to investor trades of restricted securities rather than trades involving the issuer as the seller. By using investor to investor trades he believes that he has eliminated the effects of information asymmetry, of firm financial condition and of assessment and monitoring costs since both buyer and seller should be on equal ground as far as knowledge of the firm and its financial conditions are concerned. He argues that this is the pricing concept that should be involved in fair market value where both buyer and seller are hypothetical persons and are considered to be equally knowledgeable of the conditions surrounding the trade transaction.

The LiquiStat database was compiled from April 2005 to December 2006 and contains information on 61 investor to investor trades. All investors are independent with no firm affiliates represented in any of the trades. Analysis of the data provides the following results with regard to discounts for the traded shares as compared to the present public market price of those shares.

Average Discount	32.8%
1 <sup>st</sup> Quartile Discount	19.1%
Median Discount	34.6%
3 <sup>rd</sup> Quartile Discount	44.0%

The average fraction traded in the population was 0.47%, the average stock volatility was 89% and the average remaining restriction period was 138 days.

Robak notes that these results are higher than those found by Hertzels & Smith, Bajaj *et al* and Finnerty and hypothesizes that this is because the shares studied were more volatile and had longer required holding periods than is often the case for restricted stock private placements where registration is often a condition of the placement. Per the LiquiStat data, there is a clear and increasing relationship between the discount and the remaining required holding period, although the increase tends to flatten out over time. For a 25 day holding period the discount range is 16% to 24% while for a one year holding period the discount range is 30% to 57%.

Based on his data, instead of supporting the premise that others have advanced that private placement discounts are too high to represent the effects of illiquidity alone, Robak suggests that they instead may be too low. Having eliminated the other causes of discounting through the construction of his database, he feels that the statistics derived should represent a measure of the illiquidity discount in isolation.

## **General Strengths and Weakness of Exhibit C Studies**

One of the strengths that is attributable to the analytical approach to DLOM and DLOL is that the studies underlying the approach were not made for tax purposes or by practitioners in the generally recognized tax valuation community. They were instead primarily academic exercises aimed at better understanding operating company capitalization choices and effects. Thus, there was generally no attempt made to either support or criticize the traditional benchmark study approach or to promote a new approach that could be marketed for application in tax service engagements.

A second strength is the attempt to pursue the question in an analytical manner and to parse the discounts found into contributing components. Although overall discounts were found and summary statistics identified, the researchers were more interested in the causes of the discounts than in the pure size of the discounts. Various theses were advanced as to why discounts exist with DLOL and DLOM only being one set of contributing factors. Also mentioned were assessment costs, monitoring costs, management entrenchment motives, investor expectations and certification compensation. The attempt to break out actual discount portions for these components was not necessarily successful but the conclusion that all of a given measured discount may not measure simply the lack of marketability is important.

Another potential strength is that the several researchers took several different approaches to sample selection and measurement such that the effects of these parts of the discount estimation process can be considered as part of the overall discount evaluation process. A valuator can make his or her own determination based on the facts and circumstances under study as to how data selection should be handled and how discount measurement can best be made.

The studies all suffer from the same kinds of weaknesses that make the actual numerical results achieved difficult to rely upon. These weaknesses result from problems in sample selection, problems in sample point classification, problems in discount measurement point selection, problems in variable selection, problems in variable estimation and the use of certain proxy variables with a binary quantification attribute. The various models also result in less than impressive data fits as measured by such things as  $R^2$ . In many cases the number of available data points is small and large time frames are required to yield an adequate number of data points for analysis. Many of the studies do not consider factors that would seem from a common sense point of view to have significant impact on discounts such as required holding periods and the volatility of the stock as publicly traded. Finally, although the fact that the studies were not made for tax purposes was cited as a possible strength, it is also a potential weakness in that it is dangerous to apply the results of a study made for one purpose to entirely different purposes.

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