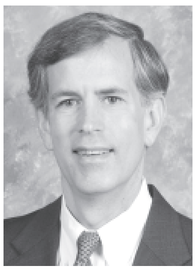
Business Valuation Review

Regression Analysis in Valuation Engagements

By: George B. Hawkins, ASA, CFA

Introduction

"Business valuation is as much as art as it is science." Sage advice, however, quantitative techniques can and should be used where appropriate in a professional valuation to arrive at more sound, logical and well supported conclusions. A case in point is the use of linear regression analysis, a statistical technique that helps discern possible relationships between two or more variables. About now you're about to tune out on the subject, suffering a bad flashback to that boring college statistics class that covered the math you thought you would never use. Wrong! This simple technique is incredibly valuable in many aspects of business valuation and also in the related issues that arise for attorneys where valuation issues come into play. These include:



George Hawkins

Identifying the factors driving the pricing paid for public companies in a particular industry and what this says about valuing the private company.

Identifying the factors driving the pricing paid for private companies in a particular industry and what this says about valuing the private company.

In equitable distribution matters in a divorce, how much of the appreciation in value of a company (that is the separate property of one spouse) over time during the marriage was due to passive versus the active efforts of the owning spouse, affecting how much of the change in value during the marriage is marital versus separate property for property division.

Regression analysis does not "prove" that there is necessarily a causal relationship between things (i.e., people that eat more Twinkies weigh more), so common sense also has to come into play. Nonetheless, it is a powerful tool with an important place in valuation and which already shapes our everyday lives now, in everything from testing new pharmaceutical drugs to making public policy decisions.

This article will not make one an expert on regression, although it will provide a basic understanding of the technique, how to interpret its results in several examples using Microsoft Excel, and provide resources to learn more about the subject.

The Basics

Regression attempts to discern relationships between things, called variables. The variable (or thing) to be predicted is the dependent variable (y), called this because its value "depends" on other independent variables (x1, x2, etc). To keep it simple, this discussion will focus on using only one independent variable. Let's take a look at a real life example to make it clear.

A valuator has been given the assignment of valuing a beer distributor and has obtained data on the prices paid in 11 transactions involving the sale of privately owned beer distributors, including the price paid and financial measures of the companies, including their annual revenues and earnings (as

(Continued on Page 2)

Regression *(continued)*

measured by earnings before interest, taxes, depreciation and amortization, or "EBITDA"). The appraiser must decide what measure(s) (the independent variables) such as revenues, EBITDA, profit margin, etc. best predict the value (the dependent variable, y) of the private company. Therefore, using Excel the analyst performs several linear regressions to test the relationship between the prices paid (y) for the beer distributors that were sold and each of the independent variables (the annual revenues, EBITDA and profit margin of each beer distributor) to discern relationships the price paid.

Shown below is a summary of the data for each acquired company, including the price paid, the price expressed as a multiple of annual EBITDA and revenues, and the annual EBITDA and annual revenue figures of each beer distributor:

Prices Paid to Acquire Beer Distributors ($ in 000s)

	Price Paid as Multiple of:			Annual
Price Paid	EBITDA	Revenues	EBITDA	Revenues
$94,769.0	7.04	1.55	$13,457.0	$61,283.0
$52,000.0	6.98	0.83	$7,448.0	$62,444.0
$48,400.0	8.23	0.79	$5,884.2	$61,508.2
$47,000.0	8.04	0.78	$5,845.6	$60,119.3
$33,740.0	11.68	0.79	$2,888.1	$42,705.0
$33,715.0	7.33	1.05	$4,600.0	$32,000.0
$21,100.0	7.39	0.79	$2,854.8	$26,674.2
$7,500.0	6.89	0.86	$1,087.9	$8,704.1
$3,700.0	27.90	0.63	$132.6	$5,901.2
$2,550.0	3.86	0.81	$661.0	$3,144.0
$1,800.0	10.86	0.57	$165.7	$3,138.7
Average	9.66	0.86		
Median	7.39	0.79		

The valuator could just simply take the easy way out and take the median multiple as a measure of the central tendency in the price paid, and in many instances that might be the right decision. However, the data above shows a wide range of multiples, so the appraiser wants to be sure he or she is making the best informed decision. Second, it would be very helpful to know which multiple is a better predictor of what buyers of beer distributorships pay in acquisitions, one based on earnings (EBITDA) or revenues. Insight into this issue will assist in deciding which multiple(s) to use or weight the most in the valuation.

Neither of these questions is easily answered from merely eyeballing the data, so let's test the relationships using simple linear regression, first "regressing" the price paid (the y, or dependent variable) against EBITDA (the x , or independent variable), then do the same thing with price compared to revenues. Therefore, our formulas are as follows, first based on earnings and then revenues:

Formula of Relationship Between Price Paid and a Selling Company's Earnings (EBITDA) in Transactions

y = a + bx
y = a + bx
where y = Company value (in $000s)- this is the price we are trying to predict
x = annual EBITDA (in thousands)- This is the earnings of the company, i.e., its earnings before interest, taxes, depreciation and amortization expense
a= slope intercept

Formula of Relationship Between Price Paid and a Selling Company's Annual Revenues in Transactions

y = a + bx
y = a + bx
where y = Company value (in $000s)- this is the price we are trying to predict
x = annual revenues (in thousands)
a= slope intercept

Regression analysis works by plotting the independent and dependent variable data from the transactions on a graph and then mathematically draws a line between through the very center of the data, minimizing the squared deviations of the data from the average. The y = a + bx formula then describes this relationship so that it might later be applied to the private company's specific results to estimate its value.

Shown below is the data on the price paid for the acquired beer distributors versus their annual earnings (EBITDA) (results in $000s):

Price Paid (y)	EBITDA (x)
$94,769.0	$13,457.0
$52,000.0	$7,448.0
$48,400.0	$5,884.2
$47,000.0	$5,845.6
$33,740.0	$2,888.1
$33,715.0	$4,600.0
$21,100.0	$2,854.8
$7,500.0	$1,087.9
$3,700.0	$132.6
$2,550.0	$661.0
$1,800.0	$165.7

(Continued on Page 3)

Regression *(continued)*

Using Microsoft Excel™, a regression analysis of the price paid (y) against the earnings (x) is performed. In Microsoft Excel 2007™ (the location and commands in Excel™ may differ for earlier versions) this is accomplished using the regression analysis tool found under "Data Analysis" on the Data toolbar. If you cannot find "Data Analysis" you will need to follow Excel™ instructions to install the Data Analysis toolpak that comes with Excel™.

Select "Data Analysis" and then the "Regression" tool. Using the mouse, click the "Input Y Range" and then highlight the cells where the data for prices paid is located in your spreadsheet. Be sure to begin by highlighting the label and the data, leaving no space in your spreadsheet between the label and the data (as in the prior table). Next do the same for the label and data in the X range for EBITDA. Next, select "Confidence Level" and put 95%, which means that you want to Excel to test that the relationships between the variables are not random with a 95% certainty (see explanation of this issue below). Also, select labels so that Excel will know that the first entry in the range for each variable is a label, as well as where the regression output should be posted (on the existing spreadsheet, etc.). Click OK and your regression results will then be produced as follows:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9876305
R Square	0.975414
Adjusted R Square	0.9726822
Standard Error	4714.4764
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	7936151449	7936151449	357.0614846	1.49724E-08
Residual	9	200036593.5	22226288.16		
Total	10	8136188043			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	2686.2721	2083.850586	1.28909055	0.22951138	-2027.725423	7400.2696
EBITDA	7.0344411	0.372270047	18.89607061	1.49724E-08	6.192307745	7.8765744

A detailed discussion of the meanings of each of the above measures under "Regression Statistics" and "ANOVA" (Analysis of Variance) is the subject of an entire statistics book. However, there are several critical statistics above that the business appraiser can quickly focus upon to draw broad conclusions about the regression.

The R square measure is a measure of the percentage of the variation in the y variable that can be explained by changes in the x variable. In our example above, the R square measure indicates that 97.5% of the variation in the prices paid for the beer distributors can be explained by variations in the earnings as measured by EBITDA, a very strong relationship. In other words, companies that make more sell for more, which makes sense.

However, just because R square is high does not prove that the relationship is statistically valid and other than by random chance. The "Significance F" measure tests whether or not the relationship is random or where a statistically significant relationship exists. In performing the regression, we selected a 95% confidence level. Therefore, if the Significance F statistic computed by Excel is less than 0.05 (for 5%) then the relationship is statistically significant (had we selected 90%, the Significance F would have to be less than 0.10 to be valid). The chart above shows that the Significance F statistic is actually negative to the 8th decimal place, or much less than 0.05; therefore, the relationship is statistically significant. Regardless of how strong the R square measure is, had the Significance F statistic been larger than 0.05 then it would not be possible to conclude that changes observed in the prices paid for beer distributors had a statistically significant relationship to earnings.

Therefore, a regression formula for predicting company value based on annual EBITDA (earnings before interest, taxes, depreciation and amortization expense) would be as follows (rounded):

Formula of Relationship Between Price Paid and a Selling Company's Earnings (EBITDA) in Transactions

y = a + bx

y = $2,686.3 + 7.0 x

where y = Company value (total value of invested capital, in thousands)- this is the price we are trying to predict

x = annual EBITDA (in thousands)- This is the earnings of the company, i.e., its earnings before interest, taxes, depreciation and amortization expense

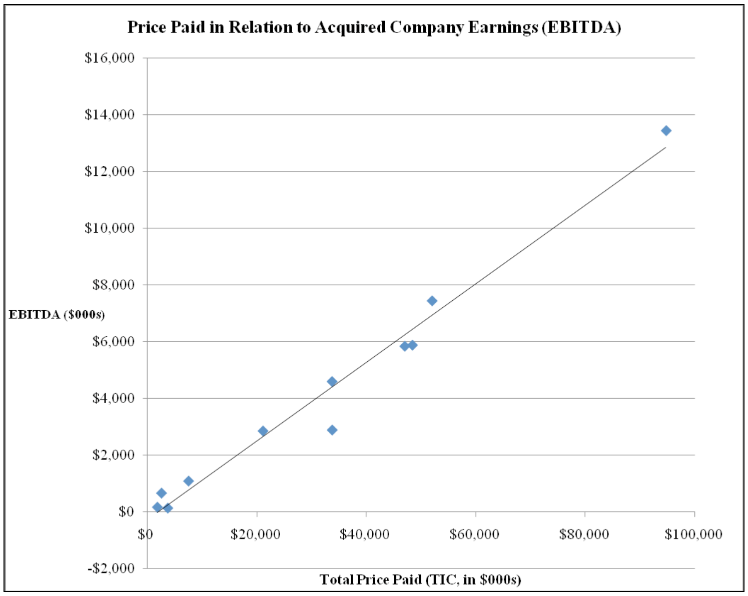
a= slope intercept

Moving beyond the math, shown below is a graphical depiction of the relationship between the price paid for the acquired company (as measured by Total Invested Capital, or TIC) versus the acquired company's level of earnings as measured by EBITDA, with the line representing the result that would be predicted by the regression formula:

Next, let's perform a similar regression of prices paid (y) against the annual revenues of the

(Continued on Page 4)

Regression *(continued)*



acquired companies. Shown below is the data on the price paid for the acquired beer distributors versus their annual revenues (results in $000s): Inputting the same data in Excel and running a simple regression provides the following results:

Price Paid (y)	Revenues (x)
$94,769	$61,283.0
$52,000	$62,444.0
$48,400	$61,508.2
$47,000	$60,119.3
$33,740	$42,705.0
$33,715	$32,000.0
$21,100	$26,674.2
$7,500	$8,704.1
$3,700	$5,901.2
$2,550	$3,144.0
$1,800	$3,138.7

In this instance, we can quickly see two things. First, the R square measure of 0.726 tells us that 72.6% of the variation in the prices paid for the beer distributors sold can be explained by changes in their level of annual revenues. While 72.6% is a relatively strong relationship, it is not nearly as powerful a predictor as with earnings (EBITDA), which explained 97.5%. Nonetheless, since the Significance F figure is less than 0.05, we can conclude that the relationship is not random and is statistically significant.

Given these results, the appraiser might then proceed to use the predicted regression formula based on earnings (EBITDA) to value the private company at issue using as follows, inserting the private company's measures to arrive at a value estimate as follows:

As shown above, using the regression formula, the estimated value for the company is $16.7 million.

Applying Merger and Acquisition Data Regression Formula to Estimate Company 100% Control Value ($000s)

Multiple applied (see previous section) (b) [1]	7.0
Times: Company EBITDA	$2,000.0
Equals:	$14,000.0
Plus: Y Intercept (a) [1]	$2,686.3
Equals: Preliminary Total Value of Invested Capital (TIC)	$16,686.3
Less: Interest-Bearing Debt [2]	$0.0
Equals: 100% Control Value of Company Equity	$16,686.3

[1] Based on regression obtained from the analysis of prior transaction data. The formula based on EBITDA is as follows: y= a + bx, y = $2,686.3 + 7.0 x.

[2] This is a simplistic example, where the price paid from the transaction data was defined as the total price including any interest-bearing debt assumed. Therefore, to arrive at the value of the Company's common equity, we have to subtract interest-bearing debt (it had none). Adjustments, if any, to be made in a real valuation will depend upon the nature of the data used.

Caution in Interpreting Results

Several major warnings are in order:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8523116
R Square	0.7264351
Adjusted R Square	0.6873544
Standard Error	15359.686
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	4385297792	4385297792	18.58807413	0.003517109
Residual	7	1651439753	235919964.7		
Total	8	6036737544			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-2960.5229	10790.56898	-0.274362079	0.791726736	-28476.16403	22555.11816
Revenues (x)	1.0200081	0.236584573	4.311388886	0.003517109	0.46057448	1.579441718

1. Regressions do not prove causality- a cause and effect relationship. A relationship could have a high R square and also be shown to be statistically significant. Nonetheless, the result could be utterly meaningless and there may in fact be no casual relationship between the independent variable and dependent variable. This is why commonsense has to also play a

(Continued on Page 5)

role in assessing the results of statistical analysis. In this instance, the results make sense- companies that have higher earnings sell for more, and this also fits well with the traditional foundation of valuation theory that the value of a company is based on the present value of its earnings or cash flow.

2. Not all relationships are necessarily linear in nature, so in those instances tools other than regression analysis may be indicated. Similarly, regressions are based on an assumption that the sample population data is "normally" distributed, i.e., like the bell-shaped curve. When the population is not normally distributed regression might or might not be appropriate depending upon the circumstances.

3. The quality, quantity and reliability of the underlying data impacts the degree to which regression analysis can be used and whether or not it gives reliable results. A practical problem in business valuation is that there is often insufficient data to have a large enough sample to be able to effectively utilize regression analysis.

4. While it is appealing to have the simplicity of a math formula to predict value, valuators must not lose sight that other non-quantitative measures and issues may be equally important in impacting value. As a simple example, the valuator finds out that the beer distributorship being valued is about to lose its exclusive right to distribute a national beer brand, the very thing that drove its revenues and earnings in the first place.

Active-Passive Appreciation Uses of Regression

A business appraiser has been engaged to value a distributor of residential wood flooring (Woodco) in a divorce for the husband who owns and runs it and who had owned the shares of the company prior to getting married, therefore being his "separate" property. The appraiser has been given the additional task of determining how much of the change in the value of the company during the marriage (from the time of marriage to the date the parties separated) was due to the active efforts of the husband, versus the amount of the change in value due to passive, external forces. Under relevant state law, even though the shares were the husband's separate property and not subject to division on divorce, the portion of the change in the value during the marriage that is a result of his active efforts is considered marital for purposes of equitable distribution. Hence, the need to isolate the impact of passive, external factors on the change in value, versus those that are active.

There are many potential factors external to Woodco that might impact its performance and its value at different points in time. The valuator's research and analysis has isolated what he believe are three passive forces affecting Woodco's results and it change value over time:

Changes in the level of residential housing starts
Changes in investor required rates of return for investing in common stocks
Changes in income tax rates, personal and corporate

Analysis of each of these passive forces will be considered by the valuator, although, for brevity' sake, this article will only examine the effect of housing starts. Since the parties were married on January 1, 1989 and were separated on December 31, 2006, the valuator will examine the relationship between housing starts and Woodco's performance over that time frame.

Housing Starts as a Passive Force2. Findings of Relationships- Housing Starts to Revenues

Based on the interview of Woodco officials as well as a review of public company filings with the Securities and Exchange specializing in the sale of various building materials, it appears that the level of single family residential housing starts is a significant external factor affecting results in a given year. This is a factor over which neither the husband nor other Woodco management has any control and is therefore a passive force. The overall level of residential new home construction is influenced by changes in interest rates, the overall economy, consumer confidence, population growth, the rate of new household formations, and other factors.

Woodco's products are ultimately used by single family homebuilders in the construction of new residential housing. Therefore, the initial working hypothesis is that it appears reasonable that an analysis might find that Woodco's revenues,

(Continued on Page 6)

earnings and value are significantly influenced by housing trends. Given this working hypothesis, the valuator sets out to objectively test these relationships using regression analysis. In using regression analysis, the goal here is to determine if annual residential housing starts is a statistically significant predictor of a) Woodco annual revenues, and b) Woodco annual earnings (measured by EBITDA), and if so, the direction and nature of those relationships.

The following sections outline the results of this analysis. The relationship of Company annual revenues to housing starts is first examined, followed by the relationship of Company profits.

Relationship of Housing Starts and Company Revenuesb. Relationship of Housing Starts and Company Revenues

Regression analysis results are shown below related to the degree to which United States residential housing starts in a given year (the independent variable, x) help predict Woodco's annual revenues (the dependent variable, y):

Relationship Between Woodco Revenues and Residential Housing Starts (1989-2006)

Year	Housing Starts (x)	Revenues ($000s)(y)
1989	1,146,300	$12,100
1990	1,081,400	$14,600
1991	1,003,400	$14,700
1992	894,900	$15,400
1993	840,400	$14,500
1994	1,030,100	$16,900
1995	1,125,600	$21,300
1996	1,198,400	$26,800
1997	1,076,300	$26,000
1998	1,161,000	$30,000
1999	1,133,600	$29,500
2000	1,271,400	$33,100
2001	1,302,500	$45,900
2002	1,230,900	$50,400
2003	1,273,200	$47,100
2004	1,358,500	$63,400
2005	1,499,000	$69,100
2006	1,610,500	$83,400

As is shown above, housing starts generally increased materially over the 1989 to 2006 time frame as did Woodco's annual revenues. The statistics shown above enable the analyst to explore if a statistical relationship might exist between the two or whether or not the results are by chance and random.

Using the same procedure as before in Excel™, a regression of the data, with y (annual revenues) as the dependent variable and x (housing starts) as the independent (and here passive) variable yielding the following results:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.910015553
R Square	0.828128306
Adjusted R Square	0.817386325
Standard Error	9082.811137
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	6359951781	6359951781	77.0926999	1.6248E-07
Residual	16	1319959330	82497458.16		
Total	17	7679911111			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*
Intercept	-84303.01836	13656.53815	-6.173088481	1.3381E-05	-113253.5858	-55352.45095	-113253.5858
Housing Starts (x)	0.10037266	0.011431647	8.780244863	1.6248E-07	0.076138651	0.124606669	0.076138651

Several preliminary conclusions can be drawn from the previous table:

Relationship Between Woodco Annual Revenues and Housing Starts is Statistically Significant- Since the F statistic is less than 0.05 this indicates there is a statistically significant relationship present. This indicates that there is a 95% confidence that the results seen would not come up randomly by chance, i.e., that there is a statistically significant relationship present.

Strong Relationship of Company Revenues to Housing Starts, With 82.8% of Variations in Woodco Revenues Explained by Changes in National Homebuilding Activity- Given that the regression is valid, the R squared statistic (called the coefficient of determination) indicates that a significant 82.8% of the variations observed in Woodco's annual revenues can be explained by changes in housing starts, a passive force over which Woodco has no control.

Positive Relationship Between Woodco Earnings and Housing Starts- The positive nature of the slope coefficient (b) indicates that there is a positive relationship between changes in housing starts and Woodco revenues. This is what might be expected, i.e., as housing starts increase the demand for

(Continued on Page 7)

Woodco's products might increase.

Relationship Between Housing Starts and Woodco Revenues Makes Commonsense- Statistical relationships do not necessarily mean there is a "causal" relationship present. Therefore, is there a valid reason why such a casual relationship would be expected to exist? The relationship is reasonable from a commonsense standpoint, as housing starts directly affect the demand for products used to build the houses under construction. Since Woodco's ability to sell its products depends on houses being built, the results make complete sense.

Formula to Predict Annual Woodco Revenues Based on Housing Starts- Therefore, a regression formula for predicting Woodco's annual revenues would be as follows (rounded):

$$y = a + bx$$
$$y = -\$84{,}303 + 0.10\,x$$

where y = revenues (in $000s)

x = annual housing starts
a= slope intercept

The valuator performs similar tests with Woodco's annual earnings and finds similar strong relationships.

In summary, the previous regression analysis showed that Woodco's results over time are materially impacted by trends in national housing start activity. Furthermore, this passive force explains a significant percentage of the variation in Woodco's results and does so with a statistical validity within a 95% level of confidence. In addition, companies in the same or similar industries indicate, in their filings with the Securities and Exchange Commission (SEC), the importance that housing activity has on their results and the demand for their products. This anecdotal information confirms the assessment by Woodco's management of the importance of housing activity and lends additional support to the credibility of regression analysis of the statistical nature of this relationship. In short, the levels of housing starts in a given year play a major role in influencing Woodco's performance. Since there is a close relationship of revenues and earnings to housing starts and earnings heavily impact the value of a business, this suggests that housing starts are a major passive force shaping Woodco's results and, therefore, value over time.

Caveats 2. Reasoning in Reaching Conclusion

It is clear that U.S. residential construction activity is a powerful and passive force over which management has no control, yet which plays an important role in Woodco's revenues and earnings in a given year and, ultimately, in its value. However, it is also true that Woodco cannot run on cruise control. Management must manage a company to achieve its results, compete effectively, develop new products and otherwise make the right decisions to benefit from national changes in demand. Therefore, despite what statistics show, it may not be reasonable to therefore assume that 82.8% of the variations in the value of the husband's shares can be explained by passive forces and is therefore separate property. The truth must incorporate that there is indeed a strong passive element, but also an important active element that means the effect may be less than 82.8%. Unfortunately, this is where subjective analysis and further inquiry by the valuator must come into play. However, this does not negate the powerful value afforded in this instance by regression analysis.

More Resources on Regression

This has been a vastly simplified discussion of regression analysis. To really understand and employ it in actual practice, it is important to understand the technique in detail, issues in its application and interpretation, and the pitfalls in using and interpreting its results. An excellent way to learn more about the topic is to read "A Second Course in Statistics: Regression Analysis," by William Mendenhall and Terry Sincich, published by Pearson Prentice Hall. In addition, quick Google search will result in numerous web sites with tutorials on the use and interpretation of regression analysis. ♦

George B. Hawkins, ASA, CFA, is President of Banister Financial, Inc., a Charlotte, North Carolina business valuation firm, is a member of the ASA Business Valuation Committee, and is co-author of the CCH Business Valuation Guide. This article is an adaption of a chapter from the 2007 edition (9th) of the book, provided by permission of CCH.

Contact George Hawkins: Phone: (704) 334-4932
email: ghawkins@businessvalue.com
website: www.businessvalue.com

A Note to Readers: By necessity, this article used a highly simplified example of regression analysis in business valuation, such as with active passive appreciation issues in divorce, etc. In an actual use of these techniques in a valuation assignment, other statistical techniques might also be used to test the results and larger data sets might be needed.

George Hawkins, ASA, CFA
Managing Director
Banister Financial, Inc.
Charlotte, NC
Phone: (704) 334-4932
email: ghawkins@businessvalue.com
website: www.businessvalue.com